Accuracy in ARMAMENTS

Choosing the appropriate dispersion metric to evaluate weapon and munitions precision

by SSG Douglas Ray (USA Ret.)

When a sniper is in the prone firing position on a shooting range, he is not thinking about the math behind the shot group dispersion, or distribution of the round impacts on the target in front of him. He is concerned with the "zero" of his weapon's aiming system, and with his own ability—and that of his weapon system—to put each round as close to the others and as close to the aimpoint as possible. The sniper thinks about these things in training, so that he does not have to worry about them in combat.

ON TARGET

(Previous page, left to right) Soldiers with 1st Battalion, 4th Infantry Regiment look for suspicious activity from an observation point during an area reconnaissance mission in Zabul Province, Afghanistan. (DOD photo by SPC Joshua Grenier, U.S. Army.)

Soldiers from 4th Battalion, 23rd Infantry Regiment arrive at an Afghan National Police checkpoint in Helmand Province, Afghanistan. (U.S. Army photo by TSGT Efren Lopez.)

Soldiers from 173rd Airborne Brigade Combat Team train at the Joint Multi-National Training Center in Hohenfels, Germany. (U.S. Army photo by Gary L. Kieffer.)

(This page) Scouts with the 82nd Airborne Division's 1st Brigade Combat Team fire on a line during a course in advanced rifle marksmanship March 21-24, 2011, at Fort Bragg, NC. (U.S. Army photo by SGT Michael J. MacLeod.)

Some of the statistical techniques used by armaments engineers and scientists to evaluate the accuracy and precision of weapons and munitions can result in increased risk to the warfighter. It is the responsibility of the armaments community to fully understand the statistical implications of choosing a specific dispersion metric to assess performance.

TARGET IMPACT DISPERSION MODEL

The bivariate normal impact distribution approach is used to model the dispersion of shot groups on a target, whether for small-caliber ammunition being fired at paper targets on a shooting range, artillery rounds impacting a target area on the ground, or darts being thrown at a dartboard.

When, for example, 10 rounds are fired at a target on a rifle range in what is typically referred to as a "shot group," each of their impact locations is a point (x_i, y_i) on the two-dimensional surface. Most of the rounds will tend to cluster around the center of impact (CoI), which is the average of all points in the x and y direction (x,y), and is the best estimator of the true mēān (μ_x, μ_y). If the weapon's aiming system is in zero with the ammunition, this means that the mean (μ_x, μ_y) is aligned with the point of aim such that $(\mu_x, \mu_y) = (0, 0)$. Therefore we can disregard aiming error, and for the purpose of this discussion concern ourselves only with the weapon system's precision.

Weapon system precision is defined by the expected error of an individual round, or "sigma" (σ).

As the distance from the CoI increases radially (where distance = $\sqrt{(x_i-\mu_x)^2 + y_i-\mu_y)^2}$) on the target, the frequency of shot impacts should decrease in a manner directly related to the magnitude of σ_x and σ_y, as defined by the Bivariate Normal Impact Distribution. This is similar to the way in which the distance in 'k' σ's from the mean of the normal distribution (with which many of us are already familiar) affects the frequency of data. That is, 95 percent fall within the range described by $-1.96\sigma < \mu < 1.96\sigma$. This relationship affords the analyst the ability to calculate the hit probability, or P[Hit], given a specified target size and shape in relation to (σ_x, σ_y), by calculating the area under the distribution's surface in the x and y direction spanned by the target.

MEASURES OF DISPERSION AND MINUTE OF ANGLE

An often-heard expression in long-distance shooting circles is "minute-of-angle," or MOA. A "1 MOA weapon system" refers to the ability of the weapon and ammunitions to consistently shoot three- to five-round groups that measure approximately 1 inch at 100 yards, approximately 5 inches at 500 yards, approximately 10 inches at 1,000 yards, etc. But without details about the method used to measure the shot group, we are lacking important contextual information.

Dispersion metrics commonly used to measure weapon or munition shot group precision include mean radius (MR), radial standard deviation (RSD), circular probable error (CPE), extreme horizontal and vertical spread (EHS/EVS), mean horizontal and vertical deviation (MHD/MVD), extreme spread (ES), etc.

In fact, each of these measures is directly related to σ, which we previously defined as the expected error of an individual round. For any group of shots on a target, we can calculate all of these measures simultaneously.

RESEARCH AND DEVELOPMENT

Soldiers from the 82nd Airborne Division visit the Aviation and Missile Research, Development and Engineering Center's Small Unmanned Aerial Vehicle Laboratory. (U.S. Army photo by Merv Brokke.)

$$f(x,y|\rho) = \frac{1}{\sqrt{2\pi}\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{\left\{-\frac{1}{2(1-\rho)^2}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\,\sigma_y} + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right\}}$$

BIVARIATE APPROACH

Bivariate Normal Impact Distribution, where ρ = Correlation(x, y).

APPLICATIONS OF VARIOUS METRICS

There are times when some of these dispersion metrics may be more desirable than others, depending on the intended application. The use of certain metrics relies on the assumption that $\sigma_x = \sigma_y$. This null hypothesis can be easily tested with a variation of the *F*-test that is commonly used in statistics. These measures have varying levels of statistical efficiency associated with them. Depending on available resources, some may be much easier to calculate than others.

For example, a rifleman or Soldier zeroing, or confirming zero on a shooting range, typically shoots three- to five-round groups and is concerned mainly with the difference between the aimpoint and the observed CoI. This same rifleman honing his or her marksmanship skills may be more interested in the distance between the two farthest points on the target, or ES. Though ES is one of the least efficient methods to calculate dispersion, in this case it is desired because of its simplicity; it requires no use of mathematics, and in fact does not even require (x, y) coordinates of the rounds on target. All that is needed is a straightedge to measure the two points that are farthest apart.

An entirely different application is used in ammunition Lot Acceptance Testing (LAT) for accuracy, in which a relatively small quantity of ammunition (a random sample) is pulled from a larger population (ammunition lot). The sample typically is tested in one or several rigid-mounted accuracy barrels, which are used as gauges to minimize the weapon system's influence on shot dispersion, so that the ammunition may be judged solely on its performance. Often, with small-caliber ammunition, MR is used to measure dispersion. The MR technique uses the distance formula shown above to determine the distance of each round from the CoI, and then takes the average of all of the points' radial distances.

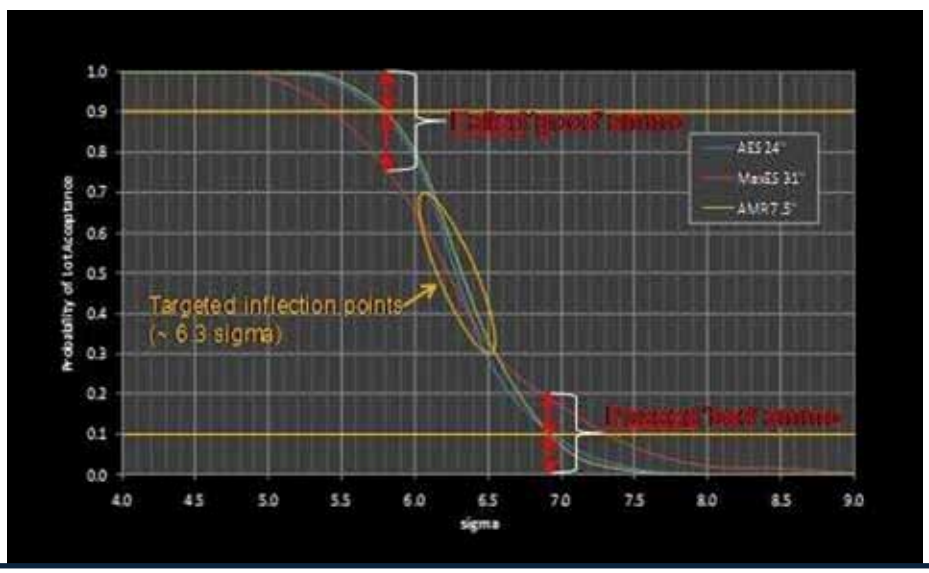
Other common methods are CPE (for some artillery and shoulder-fired rockets), RSD (the most efficient method), and EHS/EVS (for 7.62mm M118LR sniper ammunition).

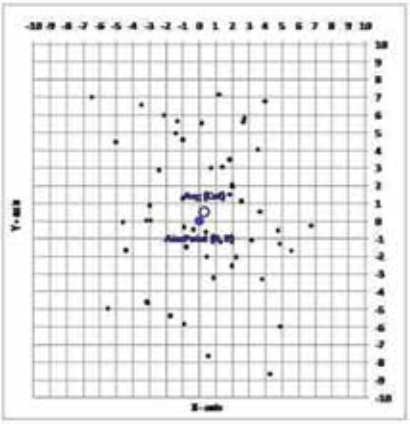
In LAT accuracy testing, the total sample quantity, breakout of rounds per target vs. number of targets in the test, and the method used to calculate the dispersion of the rounds on each target all contribute to the risks associated with accepting or rejecting lots of material.

In determining LAT quantities and acceptance criteria, operating characteristic (OC) curves are often used. They are useful to the statistician and quality engineer in that they model the probability of acceptance, or P[a], of lots of materiel, given some rate of nonconformity (or some other characteristic) within the lot. Acceptance criteria are set with the objective of rejecting lots that fail to meet these criteria ("bad" materiel), and accepting lots that meet the criteria ("good" materiel). There are, however, two other

OPERATING CHARACTERISTIC CURVE

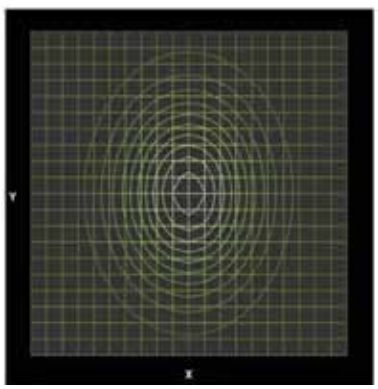
OC curve of the LAT performance difference due to dispersion metric.





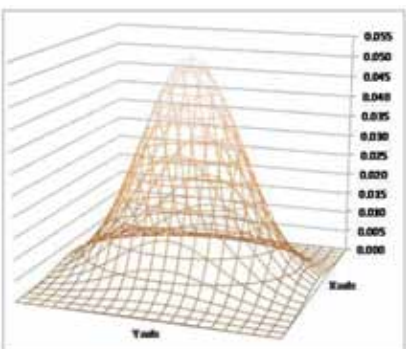
MONTE CARLO SIMULATION

Monte Carlo Simulation of a target with 50 points, CoI = (0.31, 0.51), $\sigma_x = 3$, and $\sigma_y = 4$.



BIVARIATE DISTRIBUTION

Bivariate Normal Distribution with μ_x, μ_y, and $\rho = 0$, $\sigma_x = 3$, and $\sigma_y = 4$.



OBLIQUE PROJECTION

Oblique projection of probability density of above distribution.

possibilities, which we call α and β risk: the probability of "good" materiel failing to meet the criteria (false reject), and the probability of "bad" material passing the criteria (false acceptance).

These risks may increase or decrease, depending on the test quantities and acceptance criteria used in LAT. This is what is called "discrimination."

EXAMPLE: SMALL-CALIBER AMMUNITION

For example, some recent 7.62mm M80 ball LAT data provide an idea of the past performance of this ammunition relative to its accuracy requirement. Using a mathematical relationship between the different measures of accuracy, we could select an average ES and maximum ES requirement comparable to the current LAT requirement of 7.5-inch average MR (AMR). This can be verified graphically where the inflection points (50th percentiles of P[a]) of the OC curves of each should be approximately aligned.

Keeping the sample sizes and number of targets in the LAT constant, we can perform numerous Monte Carlo simulations across a range of σ's, which will allow us to compare the discrimination of the proposed LAT requirement with the current requirement.

Comparing the OC curves of the dispersion metrics illustrates the increased risk associated with switching to a maximum ES requirement vs. averaging the targets. The risk of falsely rejecting good ammunition (α-risk) increases to ~25%, and the risk of falsely accepting bad ammunition (β-risk) increases to ~20% at the same points where AMR α = β = ~10%. This result can be deduced intuitively by understanding the nature of these different measures: Whereas ES uses only information from two rounds per target, MR uses information from all rounds and therefore is much more statistically efficient.

Over the course of numerous LATs, this increase in risk translates to excessive non-value-added production costs, reduced performance, and schedule impact. In this example, these are all due to using a different method to calculate dispersion from the same data.

CONCLUSION

When determining weapon or munition accuracy requirements or how to test for accuracy, it is important to understand not only how sample size, target and group breakout, and acceptance criteria affect the discrimination and risks inherent in any test, but also how selecting the right method to calculate target impact dispersion can affect discrimination and risk as well. This discussion has merely scratched the surface of these considerations. Armaments engineers and scientists need to address these issues in test and evaluation so that the warfighter doesn't have to deal with them in combat.

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