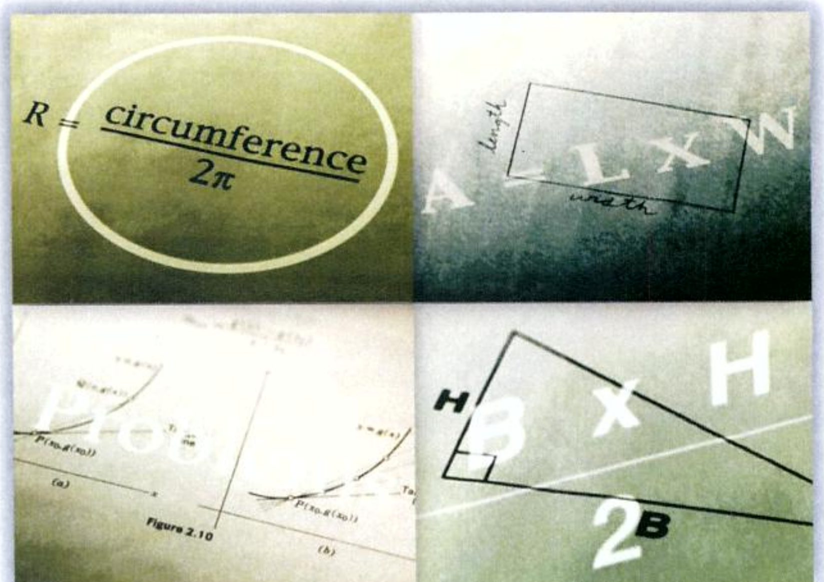
Defense Acquisition University

CON 170

Math Refresher

A Basic Mathematics Reference Guide



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CON 170
Math Refresher for Acquisition

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Block 1

Introduction to Mathematics and Excel®

Overview

Introduction

In this block, we will discuss:

- Mathematical terminology, symbols and definitions
- Basic math, algebra and statistics
- Microsoft Excel basics

Block objectives

At the conclusion of this block, you will be able to

- explain basic mathematical terminology
- apply basic mathematics
- apply basic algebra, and
- create a spreadsheet using Excel.

In this block

The following topics are located in this block:

Mathematical Terminology, Symbols and Definitions

Coefficients

Any factor of a number may be considered a multiplier of the other factors. Thus in the number, 6AB, 6 may be called the coefficient of AB or A may be the coefficient of 6B.

When there is no numerical coefficient in front of the variable(s) the coefficient is understood to be 1. For example 1XY is the same as XY.

Equations (or Formulas)

An equation is a statement that indicates the mathematical relationship among several different values and includes an equal sign. In an equation, the values that may change are called variables.

Equations are used to find a particular number based on the value of other numbers. For example; the equation for a straight line is $Y = A + BX$.

Exponents

When working with numbers it is sometimes easier to use powers to describe them.

Large numbers such as 1,000,000 can be written as 10^6 which would be 10 x 10 x 10 x 10 x 10 x 10. In this case the 6 is called an exponent of the base number 10 and in English one would say "10 to the 6th." The base doesn't have to be 10; for example: 3^2 is 3 x 3 which is 9.

When the exponent is 2 we say the base has been squared so 3^2 is "3 squared" and G^2 is "G squared."

When the exponent is 3 we say the base has been cubed. So A^3 is "A cubed."

Factors

When two or more numbers or variables are multiplied together they form a product. Thus, 3 and 4 are factors of 12 because 3 x 4 = 12 and 6 and Z are factors of 6Z because 6 x Z = 6Z.

Square roots

The opposite of an exponent, the square root of a number is the number that when multiplied by itself yields the original number. For example; the square root of 9 is 3 because 3 x 3 = 9.

The square root of a squared number yields the base of the squared number. For example; the square root of 5^2 is 5 and the square root of A^2 is A. The symbol for square roots is $\sqrt{}$.

Summation

A summation is a mathematical term for the sum of a set of values.
The symbol for summation is Σ.

Variables

A variable is the technical term for a letter which is used in place of a number. The numbers being represented by the letter may be known or unknown.

Ensuring that Your Answers are "In the Ballpark"

It's easy to make "simple" mistakes when performing mathematical calculations. These simple mistakes can lead to disastrous results. Many simple mistakes can be caught before they do any damage if you use some simple techniques to check and see that your answers are "in the ballpark." Some of the most useful techniques are illustrated below. Use them to check your answers before you base any decisions on them!

Meet the Professor! After the brief introduction to each topic, you'll find an icon of "**The Professor**" that is shown to the left. Wherever you see "The Professor" you will find a rule or guideline to follow. Read each rule carefully.

Since math involves many concepts, and concepts are best learned with the aid of examples, we have included some examples under each topic to show how "The Professor's rules" are applied. Sometimes there are practice exercises regarding the Professor's rules for you to do to make sure you understand what we're talking about.

The Professor Says:

When **adding** a column of numbers, the answer must be **larger** than any number in the column.

When **subtracting** one number from another, the answer must be **smaller** than the larger number.

Examples

Addition: 20 + 2 + 2,000 + 200 = 2,222. The answer must be **larger** than 2,000 which is the largest number in the series.

Subtraction: 2,000 – 200 = 1,800. The answer must be **smaller** than 2,000 which is the larger number.

The Professor Says:

Use the table below to check your multiplication:

When you multiply by	your answer must be	Examples
a number **greater** than 1	larger than the number being multiplied.	1. 200 • 250 = 50,000 2. 2 ½ • 3 = 7 ½ 3. 2.5 • 30 = 75
a fraction **less** than 1	smaller than the number being multiplied.	1. 200 • ½ = 100 2. ½ • ¼ = ⅛
a decimal **less** than 1	smaller than the number being multiplied.	1. .5 • 200 = 100 2. .25 • 5.5 = 1.375

The Professor Says:

Use the table below to check your division:

When you divide by	your answer must be	Examples
a number **greater** than 1	**smaller** than the number being divided.	1. 200 ÷ 50 = 4 2. 200 ÷ 2 ½ = 80 3. 200 ÷ 2.5 = 80
a fraction **less** than 1	**larger** than the number being divided.	1. 2 ÷ ½ = 4 2. 2 ½ ÷ ¼ = 10
a decimal **less** than 1	**larger** than the number being divided.	1. 21 ÷ .75 = 28.0 2. 2.5 ÷ .5 = 5.0

Practice

Estimate the answers to the following. Circle the best choice for each question.

1. If you add 2.356 + 121 + 78.257 + 17 + 21,989 + 9,742, your answer must be:
   a. Greater than 21, 989
   b. Greater than 78,257
   c. Less than 100,000

2. 34% (.34) of (times) 12,765 is:
   a. Less than 12,000
   b. More than 12,000
   c. More than 36,000

3. 24 ÷ ¾ is:
   a. Less than 24
   b. More than 24

Basic Mathematics—Symbols and Their Meanings

Mathematic symbols and their meanings

Listed below are some of the more frequently used mathematics symbols and what they mean.

The Professor Says:

Use the chart below to interpret these mathematics symbols.

Symbol	Term	What it Means
$a + b$	Plus	Add the two numbers.
$a - b$	Minus	Subtract the second number from the first.
$a \bullet b$, $a\,x\,b$, ab, or a(b)	Times	Multiply one number times the other.
$a \div b$, or a/b	Divided by	Divide the first number by the second.
$a = b$	Equals	The two numbers are equal in value.
$a \neq b$	Does not Equal	The two numbers are not equal in value.
$a \approx b$	Approximately equal	The two numbers are approximately equal in value.
$a < b$	Less than	The first number has a lesser value than the second.
$a > b$	Greater than	The first number has a greater value than the second.
$a \leq b$	Less than or equal to	The first number has a value that is less than or equal to the second.
$a \geq b$	Greater than or equal to	The first number has a value that is greater than or equal to the second.
a!	Factorial	The product of all whole numbers from 1 to a.
a^2	a squared	Multiply a times a.
$\sqrt{a}$	Square root	The number which multiplied by itself equals a.
$\sum a$	Sum or summation	The sum of all of the numbers that a can equal.

Examples

Restate the following mathematical phrases as English phrases.

1. $x < y$ — The value of x is less than the value of y.
2. $15 \bullet 10$ — Multiply 15 times 10.
3. $25 \neq 30$ — The number 25 does not equal the number 30.

Perform the following calculations.

1. 32/4 — Answer: 32 divided by 4 equals 8.
2. 5! — Answer: 5 factorial means $5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 120$.
3. $\sum$ n when n = 2, 4, 6, and 8

   Answer: The sum of all the numbers that n can equal. $2 + 4 + 6 + 8 = 20$

Practice

1. Restate $x \geq y$ as an English Phrase.
2. $10 \bullet 13$
3. Calculate 4!

Questions

After we review the Examples, restate and solve the following English phrases using mathematical symbols:

A) Four squared equals

B) The square root of thirty six equals

C) The square root of ten is approximately (round to the nearest tenths)

D) The summation of the numbers one thru seven equals

E) Solve for Y using the given data set: $Y = \frac{\sum_{i=0}^{n} Xi^2}{n-1}$

X_1	5
X_2	6
X_3	4
X_4	8
X_5	3
X_6	1

Basic Mathematics—Rounding

Rules for Rounding

There are many instances when it is necessary or helpful to round a decimal value to a value that doesn't include the quantity following the decimal point or to a value that contains fewer decimal places. For example, if you want to know the average number of items that were requisitioned over a given period of time, you would probably want to round your answer since you can't requisition only part of an item.

Sometimes, when you want a certain amount of precision, you may want to round to a particular decimal place, such as tenths, hundredths, or thousandths. Other times, when you only want a general estimation of a value, you may want to round to the nearest whole number, or even to the closest ten, hundred, thousand, etc.

The Professor Says:

Here are the basic steps for rounding:

Step 1. Identify the number in the position you are rounding to. Then look at the number **to the right** of that number.

Step 2. Round the number following these rules:
  a. If the number to the right is **5 or greater**, increase the number in the rounding position by 1 (round **up**).
  b. If the number to the right is **less than 5**, leave the number in the rounding position alone (round **down**).

Step 3. Then complete the rounding as follows:
  a. If you are rounding to a decimal place, drop all numbers to the right of the number in the rounding position.
  b. If you are rounding to a whole number, replace all numbers between the rounding position and the decimal place with zeroes, and drop all decimal places.

Examples

1. Round 14,549.436 to the nearest hundredth.
   Rounding to the nearest hundredth means rounding to two decimal places.

   Hundredths place is round position ↓

   14,549.436

   ↑ Look at this number -- round **up** Answer: 14,549.44

2. Round 14,549.436 to the nearest tenth.
   Rounding to the nearest tenth means rounding to one decimal place.

   Tenths place is rounding position ↓

   14,549.436

   ↑ Look at this number – round **down** Answer: 14,549.4

3. Round 14,549.436 to the nearest whole number.
   Rounding to the nearest whole number means rounding to the ones place.

   Ones place is rounding position
   ↓
   14,549.436
   ↑
   Look at this number – round down

   Answer: 14,549

4. Round 14,549.436 to the nearest ten.

   Tens place is rounding position
   ↓
   14,549.436
   ↑
   Look at this number – round up

   Answer: 14,550

5. Round 14,549.436 to the nearest hundred.

   Hundreds place is rounding position
   ↓
   14,549.436
   ↑
   Look at this number – round down

   Answer: 14,500

6. Round 14,549.436 to the nearest thousand.

   Thousands place is rounding position
   ↓
   14,549.436
   ↑
   Look at this number – round up

   Answer: 15,000

The Professor Says:

Be careful. Do not do chain rounding.

Example

Round 436.449 to the nearest whole number.

Chain rounding would round 436.449 to 436.45 and then to 436.5 and then to 437. This is not correct. Instead, round only based on the number closest to the number you want to round to. In this case the first 4 after the decimal point is the key; therefore, the answer is 436.

Practice

1. Round 53.55 to the nearest whole number.
2. Round 53.55 to the nearest tenth (one decimal place).
3. Round 53.446 to the nearest whole number.
4. Round 654.9 to the nearest ten.
5. Round 654.9 to the nearest hundred.
6. Round 44,757.0088 to the nearest thousand.
7. Round 44,757.0088 to the nearest thousandth (third decimal place).

Questions

After we review the Examples, answer the following questions.

A) Round 14.68 to the nearest tenth

B) Round 3,245,961.14 to the nearest thousand

C) Round 106.49 to the nearest whole number

D) Round 2,679.0235682 to the nearest thousandths

E) 256.3625 to the nearest hundredths

Basic Mathematics - Keeping Track of Decimal Places

It is sometimes confusing to remember what to do with the decimal point when performing calculations. If you are using a calculator, the calculator will keep track of the decimal point for you; however, if you must perform a calculation by hand, then follow the rules below.

The Professor Says:
Adding and subtracting numbers with decimal points

When adding and subtracting numbers containing decimal points always keep the decimal point in a straight line. It is easiest if you write the numbers in a column.

Example

Add the following numbers: 123 + 45.3 + .66 + 75.085 + 16.4

$$\begin{array}{r} 123.000 \\ 45.300 \\ .660 \\ 75.085 \\ \underline{16.400} \\ 260.445 \end{array}$$

The Professor Says:
Multiplying numbers with decimal points

When multiplying numbers with decimal points, **add** the total number of decimal places in both numbers and place **that many** decimal places in the answer.

Example

Multiply 23.42 by 13.356

23.42 • 13.356 = 312.79752

There are **2** decimal places in the first number and **3** decimal places in the second number; therefore, there are **5** decimal places in the answer.

The Professor Says:
Dividing numbers with decimal points

When dividing numbers with decimal points, count the number of decimal places in divisor (the number outside the division box) and move the decimal point in the dividend (the number inside the box) that many places to the right. Then place the decimal point directly above, in the quotient (the answer).

Example

Divide 15.288 by 14.56

```
            1.05
14.56 ⟌ 15.288
   vv      vv
   2
```

Practice

1. Subtract 2.6678 from 56.57
2. Multiply 2.667 • 56.57
3. Divide 56.4186 by .266

Basic Mathematics—Converting Fractions, Decimals, and Percents

Converting Fractions, Decimals, and Percents

There are many ways of representing values. Three ways are to represent them as fractions, decimals, or percents. We often need to convert from one of these representations to another. The simple rules for performing these conversions are given below.

The Professor Says:
Converting fractions to decimals

To convert a fraction to a decimal, simply divide the numerator (the number on top) by the denominator (the number on the bottom) and insert the decimal point in its proper place. You will probably want to round your answer to two places (nearest hundredth).

Examples

1. Convert 7/10 to a decimal.
   Answer: $7 \div 10 = .7$

2. Convert $\frac{75}{353}$ to a decimal and round the answer to the nearest hundredth.

   Answer: $75 \div 353 = .2124645$ or .21

The Professor Says:
Converting decimals to fractions

To convert a decimal to a fraction, remove the decimal point and write the number in the numerator divided by 10, 100, 1,000, etc. (depending on how many decimal places there are), and reduce the fraction. Note: To reduce a fraction to its lowest terms, find the largest number that will divide evenly into both the numerator and denominator, and divide them both by that number.

Examples

1. Convert .25 to a fraction.

   .25 is 25 hundredths

   Answer: $.25 = \frac{25}{100} = \frac{1}{4}$

   (25 is the largest number that divides evenly into both 25 and 100.)

2. Convert .537 to a fraction.

   .537 is 537 thousandths

   Answer: $.537 = \frac{537}{1{,}000}$

   (This fraction cannot be reduced.)

3. Convert 1.5 to a fraction.

   .5 is 5 tenths

   Answer: $1.5 = 1\frac{5}{10} = 1\frac{1}{2}$

The Professor Says:
Converting decimals to percents

To convert a decimal to a percent, simply move the decimal point 2 places to the **right** and add a percent (%) sign.

Examples

1. Convert .7 to a percent.	Answer: .7 = 70%
2. Convert .713 to a percent.	Answer: .713 = 71.3%
3. Convert 1.75 to a percent.	Answer: 1.75 = 175%

The Professor Says:
Converting percents to decimals

To convert a percent to a decimal, reverse the previous procedure, i.e., move the decimal point 2 places to the **left** and remove the percent (%) sign.

Examples

1. Convert 25% to a decimal.	Answer: 25% = .25
2. Convert 66.7% to a decimal.	Answer: 66.7% = .667
3. Convert 110% to a decimal.	Answer: 110% = 1.1

Practice

1. Convert $\frac{62}{553}$ to a decimal and round your answer to two decimal places.
2. Convert 7.15 to a fraction reduced to lowest terms.
3. Convert .715 to a percent.
4. Convert 2.36% to a decimal.

Questions

After we review the Examples, answer the following questions.

A) Convert 5/2 to a decimal

B) Convert 257/0.3 to a decimal (round to the nearest hundredths)

C) Convert 16/5 to a decimal

D) Convert 0.4 to a fraction

E) Convert 0.62 to a fraction

F) Convert 0.125 to a fraction

G) Convert 0.45 to a percent

H) Convert 0.055 to a percent

I) Convert 2.65 to a percent

J) Convert 95% to a decimal

K) Convert 150% to a decimal

L) What is 8% of 25,000?

M) What is 1.2% of 14,000?

Basic Mathematics - Working with Ratios

The term, ratio, is frequently used whenever mathematical comparisons are involved. For example, in a distribution depot there is a standard that says the ratio of storage locations to the number of National Stock Numbers (NSNs) stored should not exceed 3:1 (read 3 to 1).

The Professor Says:

Ratios can be written either using a colon to separate the quantities being compared or as a fraction. To express any comparison as a ratio, first write the numbers as a fraction, and then reduce the fraction to its lowest terms.

Example

1. Express $\frac{28}{36}$ as a ratio. Answer: $\frac{28}{36} = \frac{7}{9}$ (dividing both numerator and denominator by 4) or 7:9 (read 7 to 9)
2. What is the ratio of men to boys if there are 15 boys and 3 men? Answer: $\frac{3 \text{ men}}{15 \text{ boys}} = \frac{1 \text{ man}}{5 \text{ boys}}$ or 1:5 (read 1 to 5)

The Professor Says:

Sometimes you need to apply a goal or a standard that is expressed as a ratio to determine the specific ratio for a particular situation. For example, you may have a standard of 1 computer for 4 employees; therefore, you may well need to use this standard to determine how many computers you will need for an office of 80 employees. In this case, you will know one value in your ratio (80 employees), but you will need to use the standard ratio to determine the other value in your ratio (the number of computers you will need, in this case 20).

To apply a standard ratio to determine the missing number for a particular situation, follow these steps:

Step 1. Divide the known quantity by its corresponding value in the standard ratio.

Step 2. Multiply the result from Step 1 by the other value in the standard ratio.

Examples

1. If the DLA standard of supervisors to employees is 1:12, and if an organization has 360 employees, how many supervisors should it have?

   The standard is 1 supervisor : 12 employees.

   Step 1. 360 employees ÷ 12 employees = 30

   Step 2. 30 • 1 supervisor = 30 supervisors

2. A certain solvent should be mixed with water in a ratio of 2:3. How many gallons of water should be added to 12 gallons of solvent?

   The standard is 2 gallons of solvent : 3 gallons of water.

   Step 1. 12 gallons of solvent ÷ 2 gallons of solvent = 6

   Step 2. 6 • 3 gallons of water = 18 gallons of water

Practice

1. Express $\frac{7}{49}$ as a ratio.
2. What is the ratio of issues to receipts if there were 64 issues and 96 receipts?
3. A Supply Center issued a buy to demand standard of 3:4. If an Item Manager has 480 demands, what should his buy quantity be?

Basic Mathematics—Calculating Means (Averages)

Calculating Means (Averages)

There are many instances when it is important to know the average (mean) value of a set of numbers. In sports we are interested in one's bowling average or batting average. In supply management we may want to know average demand for an item for a given time period, e.g., a month. We may also want to know the average demand for the entire year. Here is how to calculate these averages.

The Professor Says:

To calculate an average, follow these three steps:

Step 1. Add (total) all of the numbers in the set you want to average.

Step 2. Count how many numbers are in the set.

Step 3. Divide the total (sum) from Step 1 by the result of your count from Step 2.

Example

Calculate the average of the following set of numbers: 126, 134, 155, 101, 144, 138, 151, 139

Step 1. 126 + 134 + 155 + 101 + 144 + 138 + 151 + 139 = 1, 088

Step 2. There are 8 numbers in the set.

Step 3. 1,088 ÷ 8 = 136

The Professor Says:

When calculating the average over several time periods, do **not** average the average values for the time period! You must take the average of all the values in all of the time periods, following these steps.

Step 1. Find the total of **all** of the numbers in **all** of the sets you want to average.

Step 2. Determine the **total** number of numbers in **all** of the sets.

Step 3. Divide the total from Step 1 by the total number from Step 2.

Example

Given the following demand history information with the calculations of the averages for each quarter as follows:

	1st Qtr.	2nd Qtr.	3rd Qtr.	4th Qtr.	Total 4 Qtrs.
TOTAL	2,462	657	970	839	4,928
FREQ.	237	101	149	128	615
Item Manager's Calculation					
Averages	10.39	6.50	6.51	6.55	8.01

Calculate the average for the year.

Step 1. 2,462 + 657 + 970 + 839 = 4,928

Step 2. There are 615 total numbers in the set.

Step 3. The average of the averages is 8.01

Note: If you add the quarterly averages (10.39 + 6.5 + 6.51 + 6.55 = 29.95) and divide by 4 ($\frac{29.95}{4}$ = 7.49) you get an answer that is different from the average for the year.

Practice

1. Calculate the average of the following set of numbers and round your answer to the nearest tenth:

   121.7, 77.6, 19.3, 36, 106.5

2. Calculate the annual average for the data in the table below.

	1st Qtr.	2nd Qtr.	3rd Qtr.	4th Qtr.
TOTAL	716	1,079	891	918
FREQ.	96	118	88	84
Item Manager's Calculation				
Averages	7.46	9.14	10.13	10.93

Questions

After we review the Examples, answer the following questions.

A) Calculate the average of the following numbers:
18, 25, 13, 13, 21, 19, 17

B) Find the average cost of widgets for the year using the data below:

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	Total
Cost	$8,321	$9,012	8,481	$8,456	
Amount Bought	32	38	34	36	
Averages					XXXXXXXXXX

Average for the year	

C) Find the average cost of aircraft flight time per hour for the year using the data below:

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	Total
Cost	$1,000,200	$1,590,100	$2,430,400	$1,300,500	
Flight time in hrs	2,015	4,000	5,400	3,458	
Averages					XXXXXXXXXX

Average for the year	

Basic Mathematics—Percent Increases, Decreases, Changes

Percent increases, decreases, changes

Questions like, "What would we make if we increased our revenue by 25%?" or, "How much stock would remain if we decreased our current balance by 30%?" are typical in today's environment. We frequently want to know what the result would be if we increase or decrease a value by a certain percent. We also often know that a measurement has changed from one value to another, and want to answer the question, "By what percent has it changed?" It is important to keep clearly in mind just what it is that you want to know when working with percents.

We will start by reviewing how to calculate the result of a specific percent increase or decrease. It is very important to realize that we do not calculate percent increase and percent decrease the same way.

Percent Increase

The Professor Says:
Calculating Percent Increase

To calculate the result after a specified percent increase, follow these three steps:

Step 1. Convert the percent of increase to a decimal by moving the decimal point two places to the left.

Step 2. Add 1 to the result of step 1.

Step 3. Multiply the initial value by the result of step 2.

Examples

1. Find the result when 500 is increased by 30%

   Step 1. 30% = .30

   Step 2. 1 + .30 = 1.30

   Step 3. 1.30 • 500 = 650

2. An activity had revenue of $5.47M last fiscal year (FY). What will their revenue be for this FY if they reach their goal of a 6% increase?

   Step 1. 6% = .06

   Step 2. 1 + .06 = 1.06

   Step 3. 1.06 • $5.47M = $5.7982M = $5,798,200

3. Find the result when 30 is increased by 250%

   Step 1. 250% = 2.50

   Step 2. 2.5 + 1 = 3.50

   Step 3. 3.50 • 30 = 105

Questions

After we review the Examples, answer the following questions using the three steps for performing percent increase problems:

Step	Action
1	Convert from % to decimal.
2	Add 1 to the decimal value from step 1.
3	Multiply the number being increased by the value from step 2.

A) What is the result when 280 is increased by 12%?

B) What is the result when 13,456 is increased by 8%?

C) What is the result when 980 is increased by 210%?

D) DLA sold 70,461 tires in the first quarter of 2010. Next quarter a 25% increase in tire sales is expected.

How many tires are expected to be sold next quarter?

E) In November a DoD activity placed an order for $170,000 worth of Meals Ready to Eat (MRE). In December the activity increased that order by 4%.

How much did the activity pay for the December MREs?

Basic Mathematics—Percent Increases, Decreases, Changes, cont.

Percent Decrease

The Professor Says:
Calculating Percent Decrease

To calculate the result after a specified percent decrease, follow these three steps:

Step 1. Convert the percent of decrease to a decimal by moving the decimal two places to the left.

Step 2. Subtract the result of Step 1 from 1.

Step 3. Multiply the result of Step 2 times the number you want to decrease.

Examples

1. Find the result when 500 is decreased by 30%

   Step 1. 30% = .30

   Step 2. 1 – .30 = .70

   Step 3. .70 • 500 = 350

2. An activity had expenses of 5.47M last fiscal year (FY). What will their expenses be for this FY if they reach their goal of a 6% decrease?

   Step 1. 6% = .06

   Step 2. 1 - .06 = .94

   Step 3. .94 • $5.47M = $5.1418M = $5,141,800

Questions

After we review the Examples, answer the following questions using the three steps for performing percent decrease problems:

Step	Action
1	Convert from % to decimal.
2	Subtract the decimal value from 1.
3	Multiply the number being decreased by the value from step 2.

A) Find the result when 120 is decreased by 14%.

B) Find the result when 85 is decreased by 7%.

C) Find the result when 27,435 is decreased by 21%.

D) An activity had expenses of $3.76 Million last year. This year the activity hopes to cut expenses by 5%.

What will expenses be this year?

E) Last month's electric bill was $330. Next month's bill will be 3% lower.

How much will next month's electric bill be?

Basic Mathematics—Percent Increases, Decreases, Changes, cont.

Percent Change

The Professor Says:
Determining the Percent Change

When a quantity or measurement has changed from an initial value to a final value, follow these steps to find out by what percent the value has changed.

Step 1. Find the amount of the change by subtracting the initial value from the final value.

Step 2. Divide the amount of the change from step 1 by the initial value, and multiply by 100 to change the fraction to a percent.

Examples

1. The price of an item has changed from an original price of $40 to a current price of $50. What is the percent change?

   Step 1. Calculate amount of change
   Final Value – Initial value
   $50 - $40 = $10

   Step 2. Calculate percent of change
   Amount of change ÷ Initial value • 100
   $10 ÷ $40 • 100 = 25%

   Note that the change has a positive sign, reflecting a 25 percent **increase**.

2. Total orders for June of last year was $16,000. For June of this year, orders totaled $13,600. What is the percent change from last year to this?

   Step 1. Calculate amount of change
   Final value – initial value
   $13,600 - $16,000 = -$2,400

   Step 2. Calculate percent of change
   Amount of change ÷ Initial value • 100
   -$2,400 ÷ $16,000 • 100 = -15%

   Note that the change has a negative sign, reflecting a 15 percent **decrease**.

The Professor Says:

Be careful. Do not confuse a percent change with a change in percentage points.

Example

Tom's accuracy rating at the shooting match dropped from 90% last quarter to 81% this quarter. Which of these correctly describes the change?

a. Tom's rating has decreased by 9 percent.

b. Tom's rating has decreased by 9 percentage points.

Answer: B

Practice

1. Find the result when an organization's 372 member staff is increased by 15%.
2. Find the result when an organization's 372 member staff is decreased by 15%.
3. Find the percent change when an organization's staff has changed from 70 members to 105 members.
4. Find the percent change when an organization's staff has changed from 150 members to 120 members.
5. The probability of precipitation for today was changed from 40% this morning to 20% this afternoon. By how many percentage points has the probability changed?

Questions

After we review the Examples, answer the following questions using this equation:

$$\%\ change = \frac{\text{Final amount} - \text{Initial amount}}{\text{Initial amount}} \text{ X } 100$$

A) Find the percent change when an organization's 1,290 member staff is increased to 1,342.

B) Find the percent change when an organization's 281 member staff is decreased to 248.

C) An item that cost $280 last year now costs $312. What is the percent change in cost?

D) Total cost for boots in May of last year was $12.5 million. This year total costs for boots for May was $9.8 million. What is the percent change in cost of boots?

E) Last week I traveled 480 miles. This week I traveled 128 miles. What is the percent change in the miles I traveled?

Basic Mathematics - Adding, Subtracting, Multiplying, and Dividing Numbers with Signs

Sometimes numbers are written either with a plus sign (+) or a minus sign (-) in front of them. The sign indicates value. Numbers accompanied by a plus sign are greater than zero and are called *positive numbers*, and the numbers accompanied by a minus sign are less than zero and are called *negative numbers*. Normally, positive numbers are written omitting the sign. Temperature is a good example of a use of numbers with signs. A temperature of -70°F is very cold, while a temperature of +70° is very comfortable. In this topic we give some simple rules for adding, subtracting, multiplying, and dividing numbers with signs.

The Professor Says:

Follow these rules to **add** numbers with signs:

Rule 1. To add numbers whose signs are all the same, add the numbers and place the sign in front of the total.

Rule 2. To add numbers whose signs are mixed,

a. Add all of the positive numbers together.
b. Add all of the negative numbers together.
c. Ignoring the signs, subtract the smaller total from the larger total.
d. Place the sign of the larger total in front of the result.

Examples

1. Add these numbers: -16, -18, -21, -9, and -15

   All values are negative so use Rule 1. The total of these numbers is 79.

   Since the common sign is - the result is -79.

2. Add these numbers: +21, +18, -6, +7, -16, -19

   Some values include positive and negative numbers; use Rule 2.

   a. The total of the positive numbers is +46.
   b. The total of the negative numbers is -41.
   c. The difference between 46 and 41 is 5.
   d. Since 46 is larger than 41, and 46 is positive, the answer is +5.

The Professor Says:

To **subtract** numbers with signs, change the sign of the number being subtracted and follow the rules for addition.

Examples

1. Subtract +21 from +42.

   Change the sign from +21 to -21 and add +42 and -21. Since +42 and -21 have different signs, we subtract 21 from 42 and place the + sign in front of the result.

   +42 – 21 = +21

2. Subtract -37 from +26.

   Change the sign from -37 to +37 and add +26 and +37. Since +26 and +37 have the same sign, we add them and place the common sign in front of the total.

   +26 + 37 = +63.

3. Subtract -21 from -25.

   Change the sign of -21 to +21 and add -25 and +21. Since the numbers have different signs, we are to subtract the smaller from the larger. Since 25 is greater than 21, the sign will be -. Therefore -25 + 21 = -4.

The Professor Says:

To **multiply** two numbers with signs, multiply the numbers together and follow these rules:

Rule 1. If the numbers have the same sign, then the sign of the answer will be **positive**.

Rule 2. If the numbers have different signs, then the sign of the answer will be **negative**.

Examples

Multiply these numbers:

1. +18 • +21 = +378

   (The sign of the answer is + since both numbers have the same sign.)

2. -15 • -10 = +150

   (The sign of the answer is + since both numbers have the same sign.)

3. -12 • +11 = -132

   (The sign of the answer is - since the numbers have different signs.)

The Professor Says:

To **divide** numbers with signs, divide the numbers and follow the same rules as you do for multiplication:

Rule 1. If the numbers have the same sign, then the sign of the answer will be **positive**.

Rule 2. If the numbers have different signs, then the sign of the answer will be **negative**.

Examples

Divide these numbers:

1. +180 ÷ +20 = +9

   (The sign of the answer is + since both numbers have the same sign.)

2. -150 ÷ -10 = +15

   (The sign of the answer is + since both numbers have the same sign.)

3. -132 ÷ +11 = -12

   (The sign of the answer is – since the numbers have different signs.)

Practice

1. Add these numbers with signs: -34, +18, +92, +100, -53, -32, -2
2. Subtract: -181 from -201
3. Multiply: +16 • -16
4. Divide: -420 ÷ -42

Basic Mathematics - Working with Exponents

Superscripts, known as exponents, are used to denote the number of times a number is to be multiplied by itself. While exponents can be positive numbers and negative numbers as well as fractions, we will only consider positive and negative exponents.

The Professor Says:

Symbolically, x^2 (read, "x squared") means $x \bullet x$ and y^4 (read "y to the 4^{th} power") means $y \bullet y \bullet y \bullet y$. Numerically, 3^2 means $3 \bullet 3 = 9$ and 2^4 means $2 \bullet 2 \bullet 2 \bullet 2 = 16$.

Examples

1. Calculate 3^3	Answer: $3^3 = 3 \bullet 3 \bullet 3 = 27$
2. Calculate x^4 when $x = 5$	Answer: $5^4 = 5 \bullet 5 \bullet 5 \bullet 5 = 625$

The Professor Says:

Negative exponents are used to denote the number of times the reciprocal number is multiplied by itself.

Note: The reciprocal of a number is equal to 1 divided by that number. E.g., The reciprocal of $\frac{3}{4}$ is $1 \div \frac{3}{4} = 1 \bullet \frac{4}{3} = \frac{4}{3}$. The reciprocal of 5 is $\frac{1}{5}$.

<u>Symbolically</u> $x^{-1} = \frac{1}{x}$ and $y^{-3} = \frac{1}{y} \bullet \frac{1}{y} \bullet \frac{1}{y}$ and $(\frac{a}{b})^{-3} = \frac{b}{a} \bullet \frac{b}{a} \bullet \frac{b}{a} = \frac{b^3}{a^3}$

<u>Numerically</u> $4^{-1} = \frac{1}{4}$ and $2^{-3} = \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{8}$ and $(\frac{2}{3})^{-3} = \frac{3}{2} \bullet \frac{3}{2} \bullet \frac{3}{2} = \frac{27}{8}$

Examples

1. Calculate 3^{-3}	Answer: $3^{-3} = \frac{1}{3} \bullet \frac{1}{3} \bullet \frac{1}{3} = \frac{1}{27}$
2. Calculate x^{-4} when $x = 5$	Answer: $5^{-4} = \frac{1}{5} \bullet \frac{1}{5} \bullet \frac{1}{5} \bullet \frac{1}{5} = \frac{1}{625}$
3. Calculate $(\frac{3}{4})^{-2}$	Answer: $(\frac{3}{4})^{-2} = \frac{4}{3} \bullet \frac{4}{3} = \frac{16}{9}$

Practice

1. Calculate 2^5
2. Calculate x^3 when $x = 8$
3. Calculate 4^{-3}
4. Calculate $(\frac{1}{3})^{-2}$

Basic Mathematics—Order of Operations

Order of Operations

When several math operations are combined in a single calculation, it can be tricky to decide which calculations to perform first. Sometimes parentheses are used to partially specify the order of operations. Nonetheless, even when parentheses are used, it is important to follow a standard order in performing math operations – otherwise different people might obtain different results.

The Professor Says:

When you are performing a mathematical calculation containing a number of different operations, follow this order from **left to right**:

1. Simplify inside parentheses (following the order of operations in the remaining steps if necessary).
2. Raise numbers to powers.
3. Multiply.
4. Divide.
5. Add.
6. Subtract.

Examples

1. Simplify: $3 + 6\,(2^2 + 5) - 8 \div 2$

$3 + 6\,(4 + 5) - 8 \div 2$	Simplify inside parentheses	
$3 + 6 \bullet 9 - 8 \div 2$		
$3 + 54 - 8 \div 2$	Multiply	$6 \bullet 9$
$3 + 54 - 4$	Divide	$8 \div 2$
$57 - 4$	Add	$3 + 54$
53	Subtract	$57 - 4$

2. Simplify: $48 \div 3\,(7 - 9)^2 + 6 \div 2$

$48 \div 3\,(-2)^2 + 6 \div 2$	Simplify inside parentheses	
$48 \div 3 \bullet 4 + 6 \div 2$	Raise number to powers	
$48 \div 12 + 6 \div 2$	Multiply	$3 \bullet 4$
$4 + 3$	Divide	$48 \div 12$ and $6 \div 2$
7	Add	$4 + 3$

Practice

1. Simplify: $75 - 2\,(3^2 - 4)^2 - 25 \div 5$
2. Simplify: $240 \div 6 \bullet 2\,(35 - 5^2) - 4 \div 2$

Questions

After we review the Examples, answer the following questions.

A) 4 x 6 + 8 =

B) 10 + 2 x 3 =

C) 11 x 3 – 8 =

D) 20 + 5 x 12 =

E) 18-9 ÷ 3 =

F) 16-9 ÷ 3 + 6 x 3 =

G) 100 ÷ 50 + 75 ÷ 25 + 1=

H) (4 + 6) x (2 – 8) =

I) 4(5 + 2) ÷ 2 =

J) 6(9 + 2) ÷ (3 x 2) =

K) (4 + 6)2 – 8 =

L) 4 – 6(2 – 8) =

Basic Algebra—Solving Equations

Evaluating and solving equations (also known as formulas)

A formula is a mathematical statement that indicates the mathematical relationship among several different values. In a formula, the values that may vary or change are called variables. Normally a formula is created to tell you how to find a particular quantity based on the values of other quantities – for example, you are probably familiar with the formula for finding the area of a rectangle: $A = L \bullet W$ (where A stands for area, L is the length of the rectangle and W is the width).

The Professor Says:
Evaluating a Formula

To evaluate a formula, follow these two steps:

Step 1. Substitute the known numerical values for the variables in the formula.

Step 2. Perform the calculation as indicated in the formula.

Examples

1. Evaluate the formula $V = s^3$ when s = 10 in

   Step 1. $V = s^3$

   $V = (10\text{ in})^3$

   Step 2. $V = 1{,}000\text{ in}^3$ (in^3 means cubic inches)

2. Evaluate the formula $D = r \bullet t$ when r = 65 mph and t = 3 hr

   Step 1. $D = r \bullet t$

   $D = 65\,\frac{\text{mi}}{\text{hr}} \bullet 3\text{ hr}$

   Step 2. D = 195 miles

   Note: See "Units of Measure" to learn how to handle measurements.

The Professor Says:
Solving for an unknown value in a formula

When you know the value of a formula, and you know the values of all of the variables except one, you can solve for the unknown value. Follow the steps below to find an unknown value in a formula.

Step 1. Substitute all known values in the formula and perform any calculations indicated.

Step 2. Add or subtract values to both sides of the formula to isolate the unknown value (along with anything it is multiplied or divided by) on one side of the = sign.

Step 3. If the unknown is multiplied by a number, divide both sides by that number. If the unknown is divided by a number, multiply both sides by that number.

Examples

1. Given the formula $y = 3a + b$, and given that $y = 11$ and $b = 2$, find the value of a.

Step 1. $11 = 3a + 2$

Step 2. $11 - 2 = 3a + 2 - 2$

$9 = 3a$

Step 3. $\frac{9}{3} = \frac{3a}{3}$

$3 = a$

2. Given the formula $a = b^2 + \frac{c}{2}$ and given that $a = 19$ and $b = 4$, find the value of c.

Step 1. $19 = 4^2 + \frac{c}{2}$

$19 = 16 + \frac{c}{2}$

Step 2. $19 - 16 = 16 + \frac{c}{2} - 16$

$3 = \frac{c}{2}$

Step 3. $2 \bullet 3 = \frac{c}{2} \bullet 2$

$6 = c$

Practice

1. Evaluate the formula $a = \frac{1}{2}c + d^2$ when $c = 2$ and $d = 4$
2. Given the formula $V = L \bullet W \bullet H$ and given that $V = 105$, $L = 3$ and $W = 7$, find H
3. Given the formula $P = 2L + 2W$ and given that $P = 10$ and $L = 2$, find W

Equation #1

After we review the Examples, answer the following questions:

$$Y = A + BX$$

A) Get X by itself in Equation #1.

B) Get A by itself in Equation #1.

C) Solve for Y in equation #1 if A = 5, B =3, and X = 10

D) Solve for X in equation #1 if Y = 121, A = 7, and B = 2

E) Solve for A in equation #1 if Y = 512, B = 3 and X = -10

Basic Algebra—Solving Equations, Continued

Equation #2

After we review the Examples, answer the following questions:

$$I^2 \times R = W$$

F) Get R by itself in equation #2.

G) Get I by itself in equation #2.

H) Solve for W in equation #2 if I = 3 and R = 6

I) Solve for R in equation #2 if I =4 and W = 160

J) Solve for I in equation #2 if R =10 and W = 640 (There are two answers.)

K) Solve for I in equation #2 if W = 405 and R = 5 (There are two answers.)

Basic Algebra—Solving Equations, Continued

Equation #3

After we review the Examples, answer the following questions:

$$Y = \frac{A}{B} - W$$

L) Get A by itself in equation #3.

M) Get W by itself in equation #3.

N) Solve for Y in equation #3 if A = 15, B = 4, and W = 13

O) Solve for A in equation #3 if Y = 120, B = 11, and W = 3

P) Solve for W in equation #3 if Y = 45, A = 12, and B = 3

Q) Solve for B in equation #3 if Y = -1, A = 10, and W = -4 (Get B by itself first)

Basic Algebra—Solving Equations, Continued

Equation #4

After we review the Examples, answer the following questions:

$$Y = \frac{A+B}{X}$$

R) Get X by itself in equation #4.

S) Get A by itself in equation #4.

T) Solve for Y in equation #4 if A = 10, B = 4, and X = 8

U) Solve for A in equation #4 if Y = 100, B = 17, and X = 5

V) Solve for X in equation #4 if Y = 30, A = 9, and B = 15

Units of Measure

Keeping Track of Units of Measure

Units of measure are important because they tell you what a quantity represents. A length of 6 is meaningless, until you know whether the unit of measure is inches, feet, centimeters, meters, etc. When you are performing calculations with quantities, it is important to understand and represent units of measure properly. Sometimes it is necessary to convert a quantity expressed as one unit of measure (for example, feet) into another unit (say, inches). Representing and keeping track of units properly can help assure that you perform conversions and calculations correctly.

The Professor Says:
Understanding and representing compound units of measure

Compound units of measure relate one measurement to another. Many common compound units include a "p" in them, where the "p" stands for "per" and the "per" indicates that the unit before the "p" stands for one measurement, and the unit after the "p" stands for another.

To write a compound unit of measure for use in a calculation, it is helpful to write the unit as a fraction, with the unit before the "p" in the numerator and the unit after the "p" in the denominator.

Examples

1. Represent a speed of 65 mph as a fraction. Answer: $65 \frac{mi}{hr}$
   (Mph relates distance covered, expressed in miles, to the time to cover that distance, expressed in hours; this compound unit tells how many miles are covered in 1 hour.)

2. Represent an engine speed of 2700 rpm as a fraction. Answer: $2700 \frac{rev}{min}$
   (Rpm relates the number of engine revolutions to the time for those revolutions to occur, expressed in minutes; this compound unit tells how many revolutions occur in 1 minute.)

The Professor Says:
Working with compound units of measure in calculations

When you are performing a calculation, it is important to keep track of the units. Cancel units that appear in both the numerator and the denominator.

Examples

1. If a truck is going 60 mph, how much distance will it cover in 6 hours?
   $6\ \cancel{hr} \bullet 60 \frac{mi}{\cancel{hr}} = 360\ mi$
   (Notice that the hour units cancel, and the answer comes out in the distance unit, miles.)

2. If there are 60 pencils in a box and you need to order 180 pencils, how many boxes will you need?
   $180\ pencils \bullet 60 \frac{pencils}{box}$
   $180\ \cancel{pencils} \bullet 60 \frac{1\ box}{60\ \cancel{pencils}} = \frac{180}{60}\ boxes = 3\ boxes$
   (Notice that the pencil units cancel and the answer comes out in boxes.)

The Professor Says: Converting one unit of measure to another

To convert a quantity expressed as one unit of measure to another unit of measure, follow these steps:

Step 1. Identify the appropriate conversion factor or factors. (Conversion factors are fractions that express the relationship between one unit and another. Conversion factors are equal to 1, since the numerator equals the denominator. Conversion factors for some common units are provided at the end of this chapter.)

Step 2. Multiply the quantity by the appropriate conversion factor or factors.

Example

1. Express 17 gallons as quarts.

   Step 1. The conversion factor will be 4 quarts/gallon.

   Step 2. $17\ \cancel{\text{gallons}} \bullet 4\ \frac{\text{quarts}}{\cancel{\text{gallon}}} = 68\ \text{quarts}$

2. Express 360 feet as yards.

   Step 1. The conversion factor will be $\frac{1\ \text{yard}}{3\ \text{feet}}$.

   Step 2. $360\ \cancel{\text{feet}} \bullet \frac{1\ \text{yard}}{3\ \cancel{\text{feet}}} = \frac{360}{3}\ \text{yards} = 120\ \text{yards}$

3. If an office gets 36 reams of paper per quarter and there are 24 reams in a case, how many cases will be required in a year?

   Step 1. The conversion factors will be 1 case/24 reams and 4 quarters/year.

   Step 2. $\frac{36\ \cancel{\text{reams}}}{\cancel{\text{quarter}}} \bullet \frac{1\ \text{case}}{24\ \cancel{\text{reams}}} \bullet \frac{4\ \cancel{\text{quarters}}}{\text{year}} = \frac{36 \bullet 4\ \text{case}}{24\ \text{year}} = 6\ \frac{\text{cases}}{\text{year}}$ or 6 cases per year

Practice

1. Express the gas mileage measurement of 28 mpg as a fraction.
2. If a car has a gas mileage of 35 mpg, how far can the car go on a full tank of 20 gallons? (Be sure to include the units for your answer.)
3. Convert 72 inches to yards.

Some Common Conversion Factors	
Units of Length	12 Inches / 1 foot
	36 Inches / 1 Yard
	3 Feet / 1 Yard
	39.7 Inches / 1 Meter
Units of Weight	16 Ounces / 1 Pound
	2,000 Pounds / 1 Ton
	28 Grams / 1 Ounce
	2.2 Pounds / 1 Kilogram
Units of Liquid Volume	16 Ounces / 1 Pint
	32 Ounces / 1 Quart
	2 Pints / 1 Quart
	4 Quarts / 1 Gallon
Units of Time	91.25 Days / 1 quarter

Adding and Subtracting "Apples and Oranges"

Before you can add two or more quantities together, or subtract one quantity from another, the quantities must first be expressed as the same units. Adding or subtracting different units will result in meaningless answers.

The Professor Says:

Adding or subtracting two quantities with different units

You cannot add or subtract two quantities that have different units! When you want to add two (or more) quantities with different units, or to subtract one quantity from another, follow these steps:

Step 1. a. If possible, convert one quantity to the same unit as the other.

b. If you cannot convert one quantity to the units of the other, find a third (more general, if necessary) unit to which you can convert both quantities, and convert both quantities to the third unit.

Step 2. Add or subtract as necessary.

Examples

1. At the beginning of the month a shop had an inventory of 6 gallons of brake fluid. The shop reports using 3 quarts of fluid during the month (but none has been received). What is the current inventory of brake fluid?

   Step 1. Use the conversion factor of 4 quarts gallon to convert the 6 gallons to quarts.

   $$6\ \cancel{\text{gallons}} \bullet 4\ \frac{\text{quarts}}{\cancel{\text{gallon}}} = 24\ \text{quarts}$$

   Step 2.

   $$\begin{array}{r} 24\ \text{quarts} \\ \underline{-3\ \text{quarts}} \\ 21\ \text{quarts} \end{array}$$

2. If a basket contains 3 oranges and 4 apples, express its contents as a single measurement.

   Step 1. Apples and oranges are different items, with no direct conversion possible. However, apples and oranges can both be considered as "pieces of fruit".

   $$3\ \cancel{\text{oranges}} \bullet \frac{1\ \text{piece of fruit}}{\cancel{\text{orange}}} = 3\ \text{pieces of fruit}$$

   $$4\ \cancel{\text{apples}} \bullet \frac{1\ \text{piece of fruit}}{\cancel{\text{apple}}} = 4\ \text{pieces of fruit}$$

   Step 2.

   $$\begin{array}{r} 3\ \text{pieces of fruit} \\ \underline{+\ 4\ \text{pieces of fruit}} \\ 7\ \text{pieces of fruit} \end{array}$$

Practice

1. Three different offices reported their inventories of duct tape as 180 inches, 20 yards, and 150 feet, respectively. What is the total inventory of tape for the three offices?
2. An office has 4 desks, 6 chairs, 3 bookcases, and 5 filing cabinets to be moved. Find a single quantity to describe the contents of this office that must be moved.

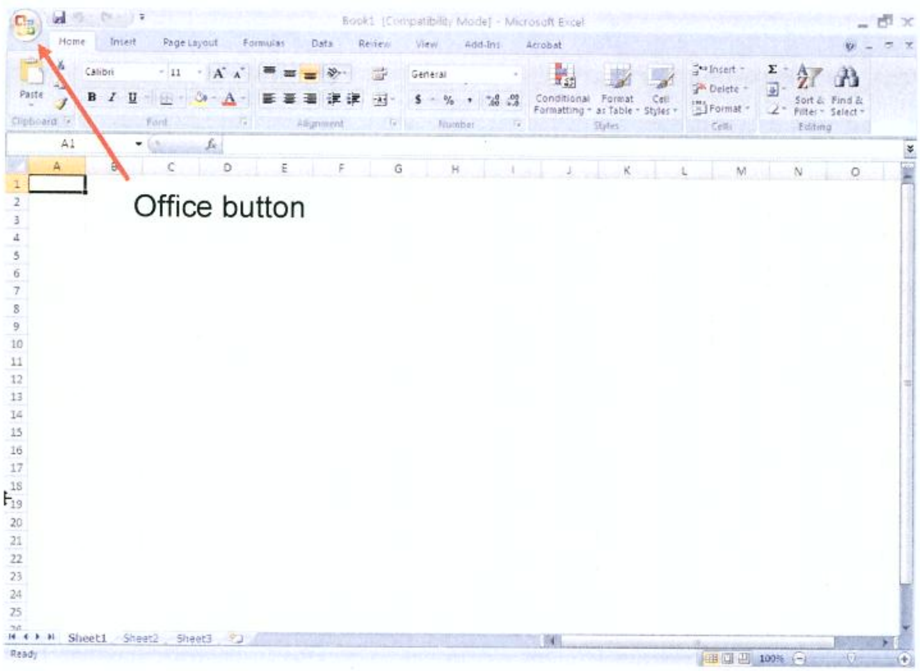
Basic Excel Information

Opening Excel

There are several ways to open Excel. The easiest is to simply double click an existing Excel file. If you don't have one available follow these steps to get Excel up and running.

Step	Action
1	Click "Start"
2	Click "Programs"
3	Click "Microsoft Office"
4	Click "Microsoft Office Excel"

Your screen should look like the one below.



Office Button

The Office Button performs many of the functions that were located in the File menu of older versions of Excel. This button allows you to

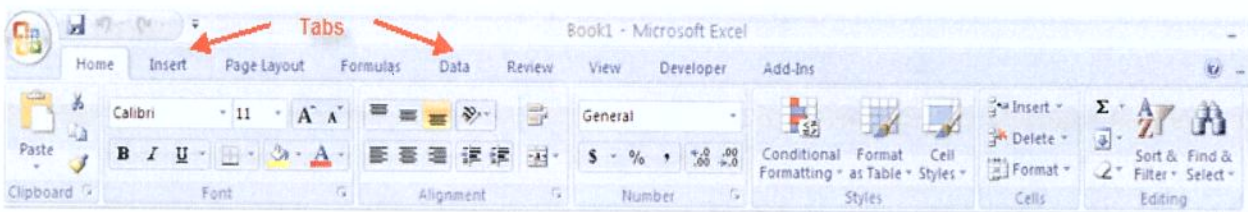
- create a new workbook
- open an existing workbook
- save
- save as
- print
- send, or
- close.

Basic Excel Information, Continued

Seven tabs in the ribbon

The ribbon is the panel at the top portion of the document. It has seven tabs:

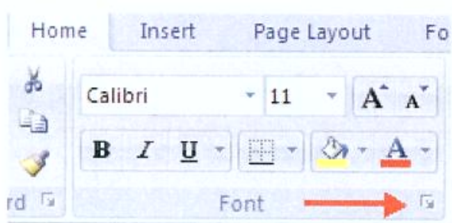
- Home
- Insert
- Page Layouts
- Formulas
- Data
- Review
- View.



Each tab is divided into groups

The groups in each tab are logical collections of features designed to perform function that you will utilize in developing or editing your Excel spreadsheets.

Commonly utilized features are displayed on the ribbon. To view additional features within each group, click the arrow at the bottom right corner of each group.



Groups within the seven tabs

Groups that are found in each of the the seven tabs on the ribbon are as follows:

- **Home**—Clipboard, Fonts, Alignment, Number, Styles, Cells, Editing
- **Insert**—Tables, Illustrations, Charts, Links, Text
- **Page Layouts**—Themes, Page Setup, Scale to Fit, Sheet Options, Arrange
- **Formulas**—Function Library, Defined Names, Formula Auditing, Calculation
- **Data**—Get External Data, Connections, Sort & Filter, Data Tools, Outline
- **Review**— Proofing, Comments, Changes
- **View**—Workbook Views, Show/Hide, Zoom, Window, Macros

Basic Excel Information, Continued

Row number

The row number is the number of the horizontal row counting from the top of the sheet.

Active Cell

Each 'box' on the Excel spreadsheet is called a Cell. The active cell will have a darker outline around it. It is the cell you have clicked on to input data.

Name Box

The name box displays the location of the currently active cell. It displays the column letter followed by the row number.

Formula Bar

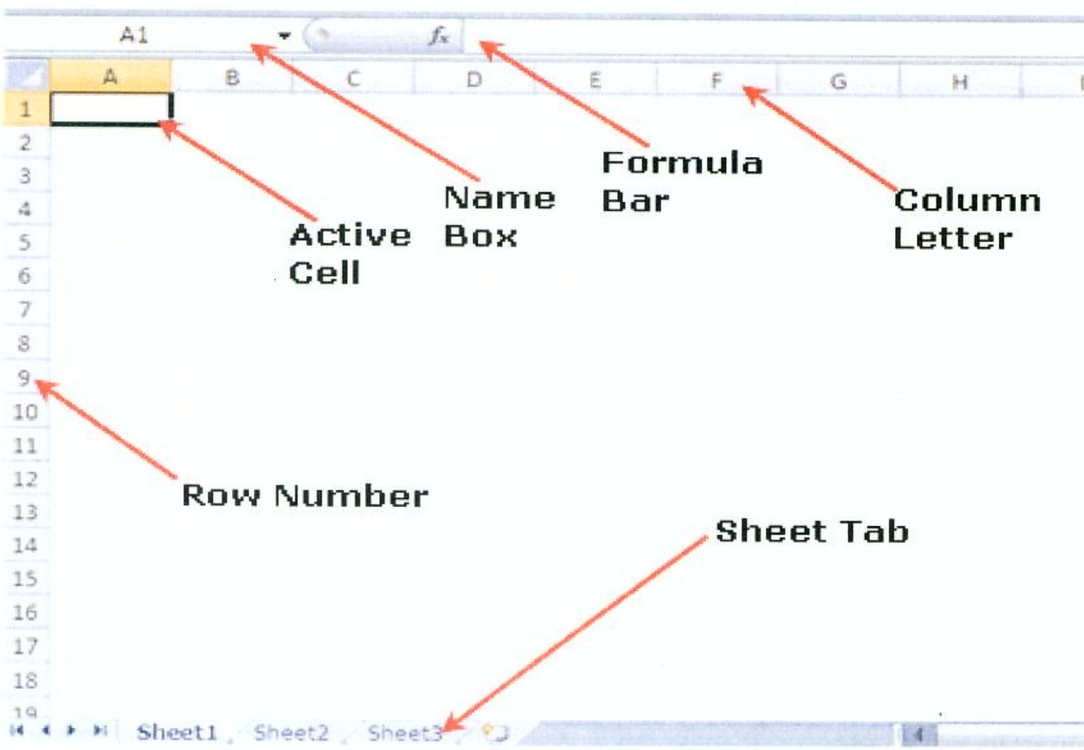
The formula bar is used to input common, premade formulas by clicking *fx* or self made formulas by typing an equal sign.

Column Letter and Row Number

The column letter is the letter designator for each column starting on the left with 'A'. The row number is the number designator for each row starting with '1' at the top.

Sheet Tab

The sheet tab denotes the name of the Excel sheet you are currently looking at what other sheets are available.



Using Excel

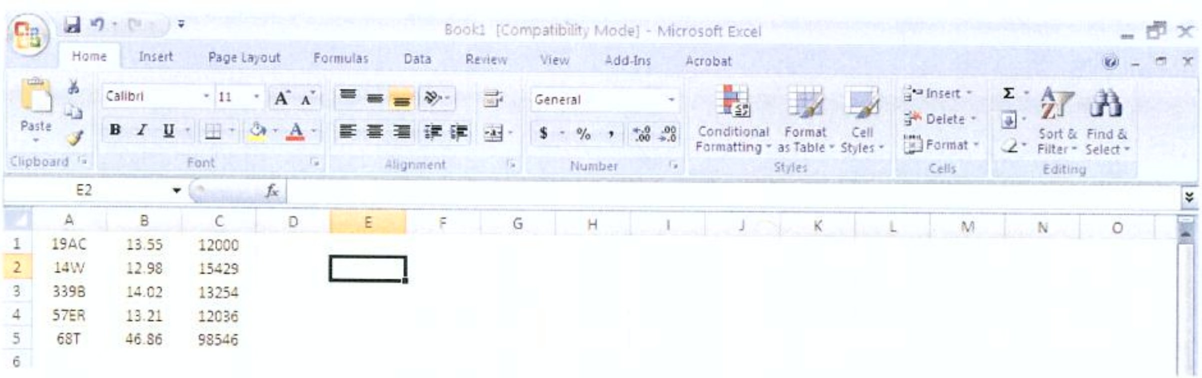
Open a new Excel spreadsheet

Use the steps we have discussed to open Excel. You should see a blank sheet on your screen. Notice that cell "A1" has a dark borderline. This is the selected cell. We will now add data and manipulate the spreadsheet.

Add data to the spreadsheet

Input the following values on the sheet starting in cell "A1".

19AC	13.55	12000
14W	12.98	15429
339B	14.02	13254
57ER	13.21	12036
68T	46.86	98546



Key strokes that help you move the cursor on the spreadsheet

The key strokes and functions listed below will help you move your cursor on the spreadsheet.

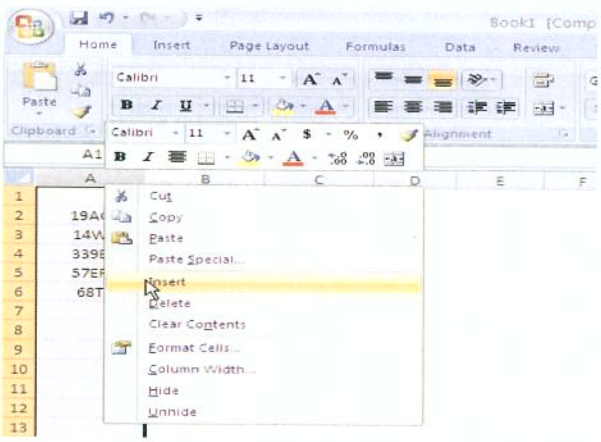
Key Strokes	Function
Tab	Moves the active cell on cell to the right.
Enter	Moves the active cell down
Holding the Shift key while using the Tab or Enter key	Moves the active cell to the left or up respectively.
Arrow keys	Moves cursor to the active cell.

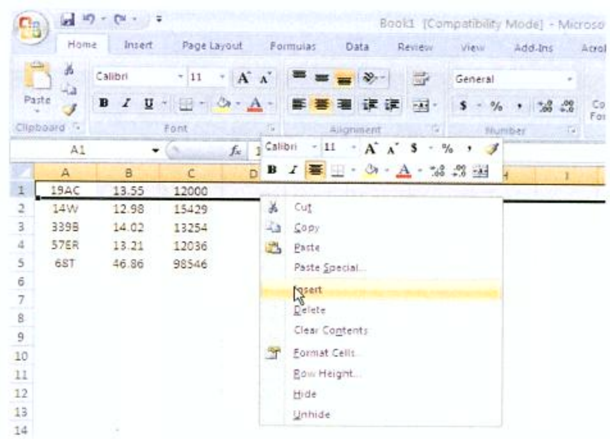
Using Excel, Continued

Adding columns and rows to the spreadsheet

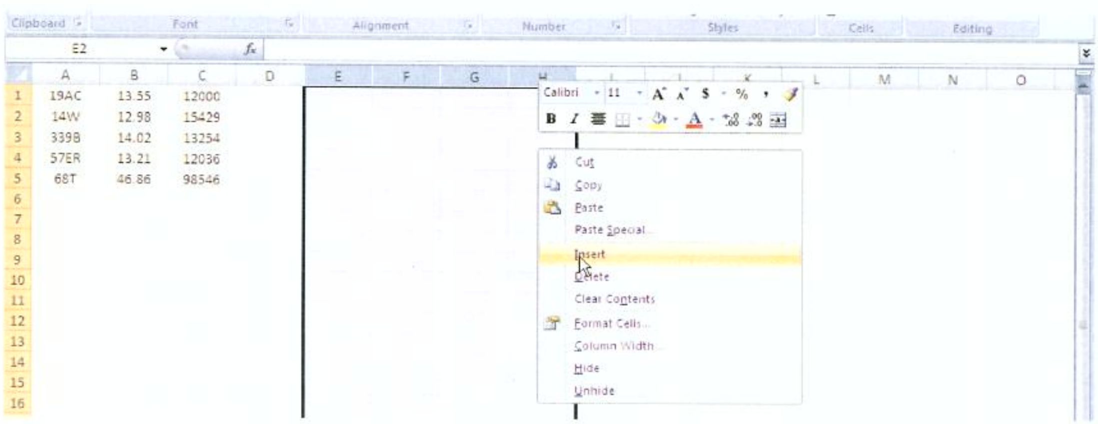
Add a new

- column to the left of column A by right clicking on the column letter and choosing "Insert"
- row above row 1 by right clicking on the row number and choosing "Insert".





Note: You can add many columns and rows at the same time by clicking on them while holding down the "Ctrl" key. Then right clicking and inserting them.

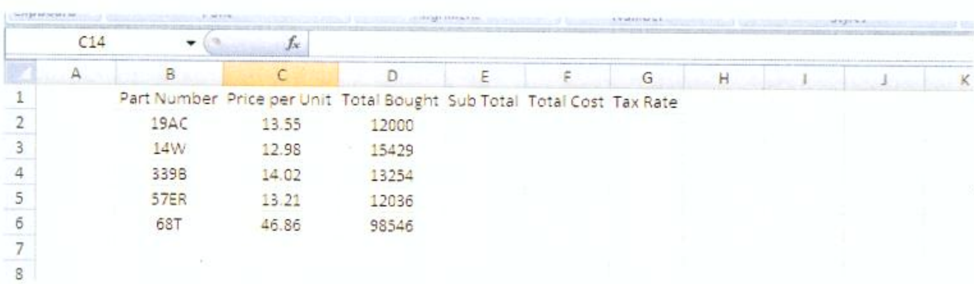


Using Excel, Continued

Add more data to the spreadsheet

Follow these steps to add more data to the spreadsheet.

In cell . . .	Input
B1	Part Number
C1	Price per Unit
D1	Total Bought
E1	Sub Total
F1	Total Cost
G1	Tax Rate



	A	B	C	D	E	F	G
1		Part Number	Price per Unit	Total Bought	Sub Total	Total Cost	Tax Rate
2		19AC	13.55	12000			
3		14W	12.98	15429			
4		339B	14.02	13254			
5		57ER	13.21	12036			
6		68T	46.86	98546			

Adjust the column widths to fit these column headers

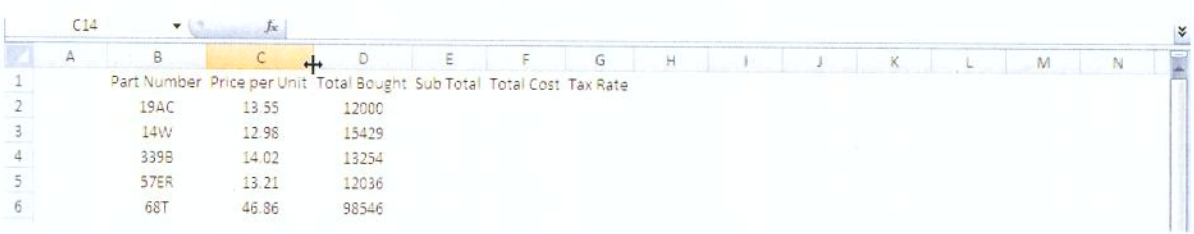
Two options for adjusting the column widths to fit these column headers are:

<u>Option 1</u>

Hover the mouse pointer over the column line to the right of the column letter and when this symbol appears left click and drag the line until the words fit

<u>Option 2</u>

Double click the to auto adjust the column width.



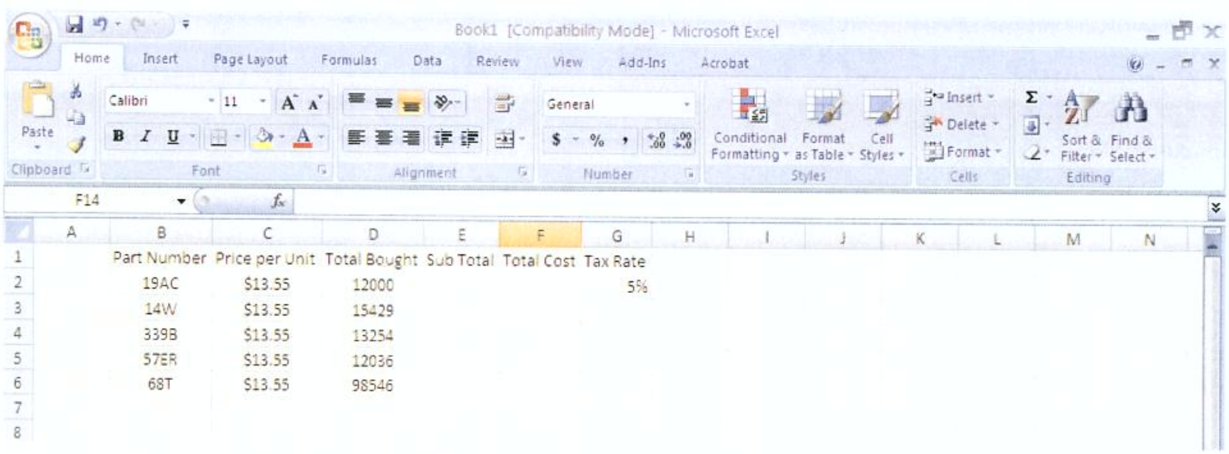
	A	B	C	D	E	F	G
1		Part Number	Price per Unit	Total Bought	Sub Total	Total Cost	Tax Rate
2		19AC	13.55	12000			
3		14W	12.98	15429			
4		339B	14.02	13254			
5		57ER	13.21	12036			
6		68T	46.86	98546			

Using Excel, Continued

Changing cell format

Change the cell format in columns C, and E to dollars and cell "G2" to percent.

Step	Action
1	Left click in cell "C2" and while holding down the left mouse button, drag the pointer to cell "C6".
2	Hold down the "Ctrl" key and left click in cell "E2" and drag down to cell "E6".
3	While still holding down the "Ctrl" key left click in cell "F2" and drag down to cell "F6".
4	On the ribbon section "Number" left click the "$"
5	Left click cell "G2" to select it and on the ribbion section "Number" left click the "%". Now type "5" in cell "G2".

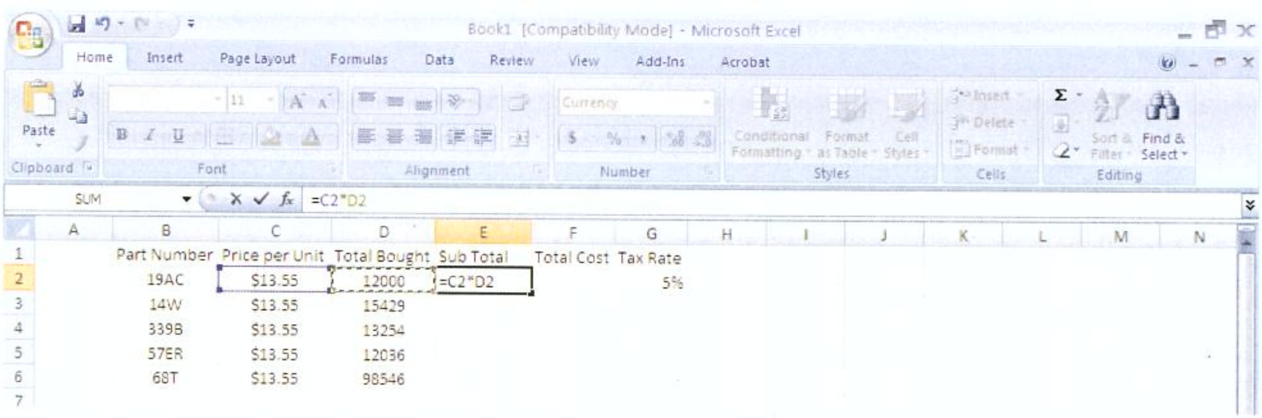


Using Excel, Continued

Calculating the Sub Total

We now will use Excel to calculate the Sub Total for each part number in column E.

Step	Action
1	Select cell "E2" and type "=". This will tell Excel you are entering a formula.
2	Click on cell "C2" and notice what happens in the formula bar.
3	Type an "*" for multiplication. Click on cell "D2". (Notice the formula bar reads "=C2*D2" Press ENTER.
4	The value that now appears in cell "E2" is the product of the values in cells "C2", and "D2" <u>Note</u>: If you change the values in these cells the Sub Total will change as well.

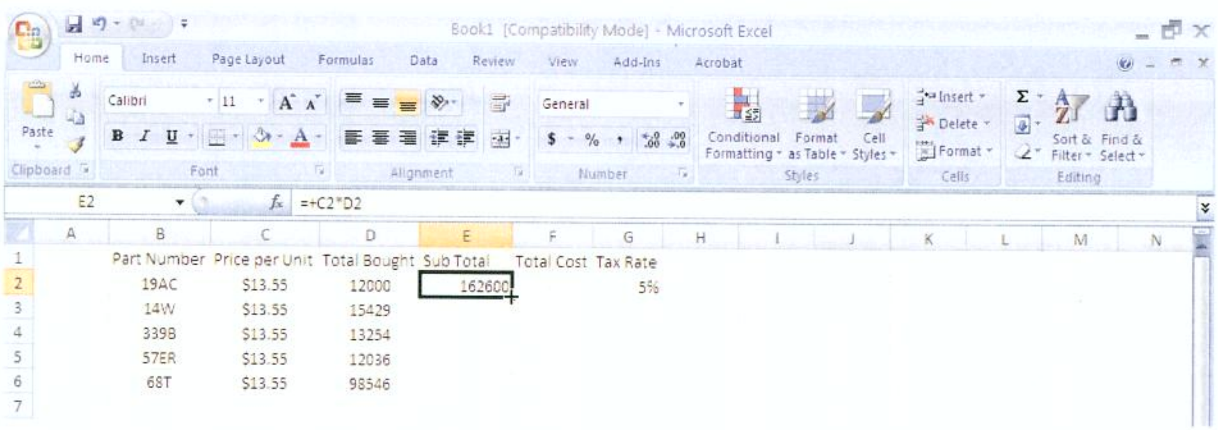


Using Excel, Continued

Calculating Sub Totals for the remaining part numbers

We now want to calculate the sub totals for the remaining Part Numbers. You can redo the steps for each row that we have just completed but there is a better and faster way.

Step	Action
1	Select cell "E2" and move the mouse pointer to the bottom right hand corner of the cell. Notice when the pointer changes from a large thick plus sign to a darker, thinner plus sign.
2	Left click the + and drag the pointer down to cell "E6." This will copy the formula in "E2" for each of the rows and the total cost will be shown for each part number. Look at each cell and notice how the cell references are changing. <u>Note</u>: **If any cell is full of "######" it means the number is too large to fit in the cell. Simply auto adjust the column to remedy this.**



Using Excel, Continued

Calculating Total Costs

We will now calculate the total cost for each part number plus tax. The Total Cost is the Sub Total + the Sub Total * the Tax Rate.

Step	Action
1	Select cell "F2" and type "=". This will tell Excel you are entering a formula.
2	Click on cell "E2" and notice what happens in the formula bar.
3	Type an + and Click on Cell "E2"
4	Type a * and enter "G2" The formula bar should read "**E2 • (1 + G2)**" <u>Note:</u> The "$" is used to lock the reference cell in the calculation. Press ENTER.
5	The value that now appears in cell "F2" is the Total Cost for Part Number A at a 5% tax rate.

Calculating Sub Totals for the remaining part numbers

We now want to calculate the Total Cost for the remaining Part Numbers. You can redo the steps for each row that we have just completed but there is a better and faster way.

Step	Action
1	Select cell "F2" and move the mouse pointer to the bottom right hand corner of the cell. Notice when the pointer changes from a large thick plus sign to a darker, thinner plus sign.
2	Left click the + and drag the pointer down to cell "F6."

Using Excel, Continued

Calculating the sum of the Total Cost

We now want to calculate sum of all the total costs.

Step	Action
1	Select "F7" to make it the active cell and click "AutoSum" found in the "Editing" section of the ribbon. Notice that Excel has selected the cells "F2" thru "F6."
2	Press the ENTER key and the summation of the total costs will be calculated.
3	Click on cell "F7" and change it to currency by clicking the "$" in the ribbon.
4	Click on the "Formulas" tab on the top of the sheet.
5	The section labeled "Formula Auditing" has a button called "Trace Precedents". Click it and arrows will appear displaying the relationships between the cells.
6	Click "Trace Precedents" again and observe.
7	Click "Remove Arrows" to remove the arrows.

Change the Tax Rate

We now want to see what effect changing the tax rate has.

Step	Action
1	Select "G2", type the number "3" and press "Enter". Notice that the values in the Total Cost column have changed.
2	Select "G2", type the number 7 and press "Enter". Notice that the values in the Total Cost column have changed.

Using Excel, Continued

Adding cell comments

Comments can be added to individual cells for later reference.

Follow the steps below to add a comment to a cell.

Step	Action
1	Right click cell "C2" and choose "Insert Comment."
2	Type "Price not confirmed".
3	When finished click anywhere outside the cell. A red triangle is now in the upper right hand corner of cell "C2" and the comment can be seen when the pointer hovers over the cell.

Editing cell comments

There may be times when you need to edit a comment.

Follow the steps below to edit a comment in a cell.

Step	Action
1	Right click cell "C2" and choose "Edit Comment"
2	In the comment box delete the old comment and type "Too expensive".

Deleting cell comments

There may be times when you need to delete a comment.

Follow the steps below to delete a comment from a cell.

Step	Action
1	Right click cell "C2" and choose "Delete Comment"
2	To undo the deletion click the "undo" button on the top left of the screen or press the "Ctrl" key and press "z" at the same time.

Using Excel, Continued

Renaming Sheets

On the bottom left of the Excel sheet you will see three tabs labeled Sheet1, Sheet2, and Sheet3. You are currently editing "Sheet1."

Follow the steps below to rename the sheet.

Step	Action
1	Right click on "Sheet1" and choose "Rename."
2	Rename "Sheet1" "Total Cost." Press ENTER.

Auto-filling Cells

Step	Action
1	Click "Sheet2" to open a fresh sheet.
2	Type "Month" in cell "A1".
3	Type "Jan" in cell "A2".
4	Select cell "A2" and move the mouse pointer to the bottom right hand corner of the cell. Notice when the pointer changes from a large thick plus sign to a darker, thinner plus sign.
5	Click the dark plus sign and drag the curser down until you see the word "Dec" in the cell
6	Enter "2001" in cell "B1" and "2002" in cell "C1"
7	Left click cell "B1" and move the mouse right until "C1" is selected as well.
8	Move the pointer to the bottom right corner of cell "C1" and click and drag to the right when it changes to a thin plus sign. Drag until "2010" is seen in the cell.

Using the "*fx*" button

Excel has several hundred commonly used formulas built in for you to use. When you click on the *fx* button on the formula bar a "Insert Function" wizard appears. You can use this to search for a function or select from a category. Click the pull down arrow in the category box and look over all the options. You can also click on the "Formulas" tab on the ribbon to see the function library.

Look around at what is available. Notice that when you hover the pointer over a function a help window appears that describes the function and how to use it.

Excel Exercises

Practice 1

You are now going to build the spreadsheets below using Excel. Click "Sheet3" and build the spread sheet below. Use formulas to calculate Total Cost, Averages and Sums.

Month	Number Bought	Unit Cost	Total Cost
Jan	15	$1.25	
Feb	26	$1.00	
Mar	20	$1.12	
Apr	16	$1.24	
May	18	$1.20	
Jun	18	$1.21	
Total			
Averages			

Add and edit some comments. Feel free to play.

Practice 2

Create a blank sheet by clicking on the "Insert Worksheet" button on the bottom of the spreadsheet next to "Sheet 3". This will create "Sheet 4". Build the spread sheet below.

Class	Number of students	Cost per student	Total Cost
A	12	$1,020.00	
B	9	$1,000.00	
C	14	$2,000.00	
D	13	$1,300.00	
E	8	$1,400.00	
F	10	$900.00	
Total			
Averages			

Add and edit some comments.

Some Key Statistical Terms and Their Meanings

Descriptive statistics are numbers that are used to describe a set of data. Statistics can help you understand an overall set of data by telling you something about how the values in the data set are spread out and where they generally lie. However, to understand a particular statistic, you need to understand what it tells you about the data. This topic introduces some basic statistical measurements.

Throughout this topic, the examples develop statistics to describe the following data identifying the number of employees hired in an office in each of the past 7 years:

1991	19
1992	37
1993	28
1994	32
1995	24
1996	19
1997	30

The Professor Says:
Find the range of a data set

The **range** of values presents the high and low values in the data set. The range gives you a general idea of where the values lie and the overall spread. The range does not give any indication of whether more values are closer to one end of the range or the other.

Example

Find the range of the sample data set.

The high value is 37; the low is 19. The range of employees hired in personnel from 1991 through 1997 is from 19 to 37 employees.

The Professor Says:
Understanding and finding the mean of a data set

The mean of a data set is the average of the values in the data set. The mean gives you a measure of the middle or center of the data, reflecting the high and low values and where the values lie in between.

To find the mean of a data set, follow these steps:

Step 1. Add all the values.
Step 2. Count the number of values.
Step 3. Divide the sum of the values from Step 1 by the number of the values in Step 2.

Example

Find the mean of the sample data set.

Step 1. 19 + 37 + 28 + 32 + 24 + 19 + 30 = 189

Step 2. There are 7 values.

Step 3. 189 ÷ 7 = 27

The Professor Says:
Understanding and finding the median of a data set.

The **median** of a data set is the value that lies in the middle of the values when all are listed from the highest to lowest or lowest to highest. Half of the values will be above the median and half will be below. The median does not reflect any measurement of how far above or below it any of the values lies.

To find the median of a data set follow these steps;

Step 1. List all data elements in order from highest to lowest or lowest to highest.

Step 2. Count the data elements and determine if the number of elements is odd or even.

Step 3. Find the median as follows:

a. If the number of elements is odd, determine which one will be the middle and count down to find that number. This is the median.
b. If the number of elements is even, determine which two values will be in the middle and count down to find those numbers. Then take their average. This is the median.

Example:

1. Find the median of the sample data set: 19, 24, 28, 32, 37, 19, 30

Step 1.

37
32
30
28
24
19
19

Step 2. There are seven elements. Seven is an odd number.

Step 3.

37
32
30
28 ← Since there is an odd number of elements, the median will be the 4th from the top or bottom.
24
19
19

2. Find the median of this sample data set: 6, 18, 32, 55, 28, 39

Step 1.

55
39
32
28
18
6

Step 2. There are six elements. Six is an even number.

Step 3. The middle values will be the third and fourth values.

55
39
32 ←
28
18
6

The median will be the mean of 32 and 28.
$(32 + 28) \div 2 = 60 \div 2 = 30$.
The median of this data set is 30.

The Professor Says:
Understanding and finding the mode of a data set.

The **mode** of a data set is the value that occurs most often. The mode does not tell you anything about how spread out the other values are or where they are relative to the mode.

To find the mode of the data set, follow these steps:

Step 1. List each value in order from highest to lowest or lowest to highest.

Step 2. Identify the value that occurs in the list the greatest number of times. This is the mode.

Example

1. Find the mode of the sample data set: 32, 37, 19, 24, 19, 28, 30

Step 1.

37
32
30
28
24
19
19

Step 2.

37	
32	
30	
28	
24	
19 ←	19 is the only value that occurs more than once; it is the mode
19	of this data set

2. Find the mode of this sample data set: 9, 9, 10, 6, 5, 6, 5, 5, 10, 4, 2, 10, 9, 9, 8, 5

Step 1.

10	6
10	6
10	5
9	5
9	5
9	5
9	4
8	2

Step 2.

	10	6	
	10	6	
	10	**5**	There are four 9's and four 5's and fewer of
	9	**5** ←	all other values. This data set is **bi-modal**;
→	**9**	**5**	that is, it has two modes, 9 and 5.
	9	**5**	
	9	4	
	8	2	

The Professor Says:
Understanding the Standard Deviation of a Data Set

Another commonly used statistic is the standard deviation. The **standard deviation** is a measurement that is used with large data sets to indicate how close most of the data elements are to the mean of the data set.

It is not unusual for a data set to have what is called a **normal distribution**, where most of the items in the data set are close to the mean and only a few are very far away from it.

If a data set has a normal distribution, two-thirds of the values in the data set will occur within one standard deviation above or below the mean. For example, if a data set has a mean of 75 and a standard deviation of 15, then two-thirds of the data elements will be between 60 and 90 (that is, between 75 - 15 and 75 + 15).

Example

> Where would you expect two-thirds of the data elements to fall for a data set that had a mean of 50 and a standard deviation of 10?
>
> Two thirds of the values would fall between 50 + 10 and 50 – 10, so two-thirds of the values would lie between 40 and 60.

Practice

Use the following set of data representing the ages of the staff in a particular office to answer questions 1 through 4.

53, 50, 45, 45, 40, 37, 37, 37, 25

1. Find the range of ages in the office.
2. Find the mean age in the office.
3. Find the median age in the office.
4. Find the mode of ages in the office.
5. If a different office had a median age of 45 and a standard deviation of 12, between what ages would you expect to find two-thirds of the employees?

Use the following data set representing the waist sizes of the people in a particular office to answer questions 6 and 7.

46, 42, 42, 40, 38, 38, 38, 36, 34, 34, 34, 32, 30, 30

6. What is the median waist size in this office?
7. What is the mode of the waist sizes in this office?

How to Convert English Phrases into Mathematical Statements

Mathematics is sometimes considered a language, because mathematical symbols can be used to represent important numerical relationships. This topic presents information to help you translate English phrases or statements about numerical relationships into mathematical statements. These mathematical statements can then be used to solve a variety of problems.

The Professor Says:
Expressing relationships in mathematical form

There are a number of common English phrases that can be directly translated into mathematical statements. These are listed in the table below, along with illustrations.

Phrase	Illustration (English) / Translation (Mathematics)
equals, is equal to	a equals b *or* a is equal to b
	$a = b$
is greater than	a is greater than b
	$a > b$
is __ more than	a is 3 more than b
	$a = b + 3$
is less than	a is less than b
	$a < b$
is __ less than	a is 5 less than b
	$a = b - 5$
is the sum of	a is the sum of b, c, and d
	$a = b + c + d$
is the difference of	x is the difference of c and d
	$x = c - d$
is the product of	i is the product of p, r, and t
	$i = p \bullet r \bullet t$ [also $i = prt$]
is the quotient of	t is the quotient of r and s
	$t = r \div s$
is __ times a much as	c is 3 times as much as d
	$c = 3 \bullet d$ [also $c = 3d$]
is __ as much as	p is half as much as q
	$p = \frac{1}{2} \bullet q$ [also $p = \frac{q}{2}$]
is the square of	c is the square of d
	$c = d^2$
equals __ factorial	c equals 3 factorial
	$c = 3! = 3 \bullet 2 \bullet 1$
Other Examples	c is 3 more than twice as much as d
	$c = 2d + 3$
	x is the sum of the squares of a and b
	$x - a^2 + b^2$
	a is 4 less than three times as much as b
	$a = 3b - 4$
	y is 5 more than one-third as much as z
	$y = \frac{z}{3} + 5$

Examples

Restate the following relationships as mathematical statements. Use letters to represent values.

1. Sue has three more vacation days left than does Jean.

   $s = j + 3$ (Where s is the number of Sue's vacation days left, and j is the number of Jean's vacation days left)

2. The number of white marbles in the pile is three times the number of black marbles.

   $w = 3b$ (Where w = the number of white marbles and b = the number of black marbles)

3. Paolo had one less than twice as many bones as Fido.

   $p = 2f - 1$ (Where Paolo has p bones and Fido has f bones)

Practice

Restate the following relationships as mathematical statements.

1. John's age is 3 years less than Patrick's age.
   (Let j = John's age and p = Patrick's age.)
2. Office a processed four times as many applications as office b.
   (Let a = the number of applications processed by office a and b = the number of applications processed by office b.)
3. The sum of the number of employees in branches a, b, and c is 3 less than twice the number of employees in branch d.

Answer Key to Practice Questions:

Page 4	1. b. Greater than 78,257 2. a. Less than 12,000 3. b. More than 24
Page 6	1. The value of x can be greater than or equal to the value of y. 2. 130 3. 24
Page 9	1. 54 2. 53.6 3. 53 4. 650 5. 700 6. 45,000 7. 44,757.009
Page 11	1. 53.9022 2. 150.87219 3. 212.1
Page 14	1. .11 2. 7 — 3. 71.5% 4. .0236
Page 16	1. 1:7 2. 2:3 or - 3. 360
Page 18	1. 72.2 2. 9.34
Page 25	1. 427.8 or 428 2. 316.2 or 316 3. +50% or 50% increase 4. -20% or 20% decrease 5. The probability has decreased by 20 percentage points.
Page 29	1. +89 2. -20 3. -256 4. +10
Page 30	1. 32 2. 512 3. — 4. 9
Page 32	1. 20 2. 0
Page 34	1. 17 2. 5 3. 3
Page 40	1. $28 \frac{miles}{gallon}$ 2. 700 miles 3. 2 yards
Page 43	1. Either 75 yards or 225 feet or 2,700 inches 2. 18 pieces of furniture
Page 62	1. 25-53 2. 41 3. 40 4. 37 5. Between 33 and 57 6. 36 7. Bi-Modal: 34 and 38
Page 64	1. J = p – 3 2. a = 4 x b or a = 4b 3. a +b + c = 2d – 3

Block 2

Introduction to Statistics

Overview

Introduction

In this block, we will discuss:

- Statistical terminology, symbols and definitions
- Basic statistics

Block objectives

At the conclusion of this block, you will be able to

- explain basic statistical terminology, and
- apply basic statistical concepts using Excel spreadsheet data.

In this block

The following topics are located in this block:

Statistical Terminology, Symbols and Definitions

Population

An entire set of elements that share one or more common characteristics. A population may have a finite number of elements such as the number of pencils in this room or an enormous number of elements such as the number of pencils on Earth. The symbol for the number of items in a population is N.

Sample

A subset of the population that is selected to make some inference about the entire population. A numerical characteristic of a sample is called a statistic. The symbol for the number of items in a sample is n.

Statistics

Calculations computed from samples to make inferences about the population. For example, if 10 contracts for tires are selected out of a population of 100 and they are evaluated for cost per tire the calculated average and standard deviation are statistics.

Confidence level or Measure of Reliability

The percentage chance that an inference about the population gained from statistics is correct. For example, we may be 95% confident that the average price being paid for an item is $2000 per unit plus or minus $200.

Confidence Interval

The range of values in which it is believed the true population mean exists. In the above confidence level, the confidence interval would be $1800 to $2200. The smaller the confidence level the more narrow the confidence interval will be.

Significance Level

The range of values that are outside the interval which is likely to contain he population mean. It is equal to 1 minus the measure of reliability. For example, if the confidence level is 95% then the significance level is 5%.

Measures of Central Tendency

The central value around which data from observations tend to cluster. The three most common measures of central tendency are the Mean, Mode and Median.

Mean

The average value of the data set. To find the mean add all the observed values and divide by the number of observations. The symbol for the mean of a population is μ. The symbol for the mean of a sample is $\overline{X}$ (pronounced "x bar").

Mode

The value of the data set that occurs most often. There can be more than one mode, or no mode at all.

Median

The middle point of a data set. The value which divides the ordered data points such that an equal number of points are below and above it.

Statistical Terminology, Symbols and Definitions, Continued

Measures of Dispersion

How much a set of data is spread out is measured by its Range, Variance, and Standard Deviation.

Range

Range is the difference between the largest observation and the smallest observation. If the largest observation is 10 and the smallest is 4 then the range is 6. Although range is the easiest measure of dispersion to calculate it doesn't describe the dispersion in detail.

Variance

The difference between an expected value and an obtained value. Variance in a process, like an unexpected outcome, is usually considered a bad thing.

Standard Deviation

The square root of the variance that measures the relative dispersion of a data set. The smaller the standard deviation, the less spread out the data is. The symbol for standard deviation for a population is σ. For a sample the symbol for standard deviation is s.

Coefficient of Variation (CV)

The measure of relative dispersion of a data set when the means of the data sets are not the same. CV is obtained by dividing the standard deviation by the mean. The CV allows the comparison of apples to oranges. The smaller the CV the less variation.

For example:

- Sample A has a mean of 25 and a Standard Deviation of 5
- Sample B has a mean of 100 and a Standard Deviation of 10

Which sample has more relative dispersion?

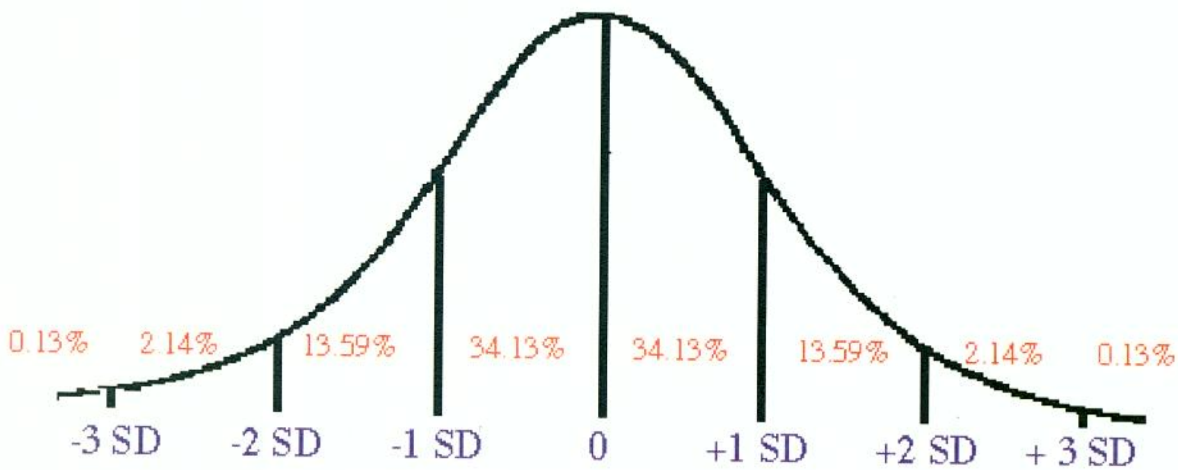
Statistical Terminology, Symbols and Definitions, Continued

Normal Distribution

Features of normal distribution include:

- Defined by the known population mean and standard deviation
- Is symmetrical
- Total area under the bell shaped curve is equal to 1.
- 99.74% of values are within 3 Standard Deviations of the Mean.

The standard normal distribution has a mean of zero and a standard deviation of one.
The area under the standard normal curve represents the probability for a given event. Tables are often used to find the area.



Normal Distribution

"t" distribution

A variation of the Normal distribution needed when the population mean and standard deviation is not known. In contract pricing, the conditions for using the normal curve are rarely met thus the t distribution is used to in pricing to establish a safe, conservative estimate of the confidence interval.

Features of "t" distribution include:

- It is symmetrical like the Normal curve but flatter and has higher tails.
- Defined by Degrees of Freedom (n-1).
- Uses a sample of the population.
- There is a different t distribution for each sample size. The t value is found on a t table.
- As sample size increases the t distribution becomes more and more like the Normal distribution.

Statistics Using Excel

Working with a pre-made Stats spreadsheet

Now that you have become familiar with Excel and some of the common Statistical terms we are going to open and use a pre-made Excel spreadsheet to answer statistical questions.

You should have available to you a spreadsheet with the file name: **Basic-Stats-Tool.xlsx.** Find this file and double click it to open it.

Input the following values in the cells underneath DATA on the left of the screen.

167	174	260
145	189	198
233	171	195
181	167	175
171	192	297
161	162	186
130	148	331
160	273	200
148	174	273
373	159	160

Answer the following questions:

A) What is the Mean of the data?

B) What is the Median of the data?

C) Is there a Mode of the data and how often does it appear?

D) What is the smallest (Minimum) number in the data set?

E) What is the largest (Maximum) number in the data set?

F) What is the range of the data set? (Maximum – Minimum)

G) What is the Standard Deviation of the data set?

H) What is the Coefficient of Variation of the data set?

I) What is the Confidence Interval if the Confidence level is 90%?

J) What is the Confidence Interval if the Confidence level is 50%?

Red data cells

Notice that three data cells are red. This means these values are more than two standard deviations from the Mean. This may require investigation to determine if the numbers should be retained, adjusted, or removed.

Statistics Using Excel, Continued

Scenario 1: Administrative Lead Time (ALT)

Your division chief wants an estimate of how much ALT will be needed on contracts next quarter that will be similar to this quarters. You decided to randomly select 30 contacts from this quarter and use their ALT's as data points.

ALT's in days

Contract 1	**13**	Contract 11	**13**	Contract 21	**12**
Contract 2	**15**	Contract 12	**16**	Contract 22	**14**
Contract 3	**12**	Contract 13	**12**	Contract 23	**14**
Contract 4	**14**	Contract 14	**15**	Contract 24	**16**
Contract 5	**16**	Contract 15	**16**	Contract 25	**19**
Contract 6	**3**	Contract 16	**89**	Contract 26	**17**
Contract 7	**12**	Contract 17	**13**	Contract 27	**15**
Contract 8	**16**	Contract 18	**14**	Contract 28	**18**
Contract 9	**18**	Contract 19	**16**	Contract 29	**15**
Contract 10	**10**	Contract 20	**14**	Contract 30	**12**

Questions

A) Using this data what do you estimate, with 80% confidence, your average ALT and confidence interval will be on next quarter's contracts?

B) Your division chief would like you to be 95% confident in your estimate. What would your answer be now?

C) Which of the two answers above would you be most comfortable with giving? Why?

D) What could you possibly do to increase the accuracy of your estimate?

E) If you research and find that contract 16 was a special case that was held up by the Product Specialist what could you do? What answer would you give your division chief with 95% confidence?

Statistics Using Excel, Continued

Scenario 2: Proposal Evaluation

The company Radar-Love is proposing to supply the government with new radar antennas. In the proposal they calculate the Engineering and Labor costs using a formula based on their engineers and laborers hourly pay. They propose to charge $81.00 per Engineering hour for design. Below is the information they provided showing the hourly pay for all their engineers.

Engineer #	Per hour
1	$65.10
2	$47.10
3	$50.80
4	$46.75
5	$130.80
6	$52.33
7	$59.50
8	$203.50
9	$49.75
10	$45.25
11	$53.00
12	$56.00
13	$60.25
14	$150.75
15	$70.00

Questions

A) Using the Basic Stats Tool, what minimum and maximum values, with 95% confidence, would you negotiate for engineering pay per hour?

B) What questions might you ask the company about the provided information? How could the answers to those questions change your negotiation values?

C) You are told that Engineer #8 is the senior engineer and that he will not be working on the project. Does this change your negotiation values? If so, what new values will you use?

D) Can you think of better information to use in order to estimate engineering costs on a contract?

Statistics Using Excel, Continued

Scenario 3: Proposal Evaluation Continued

The company Radar-Love is proposing to supply the government with new radar antennas. In the proposal they calculate the Labor costs using a formula based on their laborers hourly pay. They propose to charge $17.00 per Laborer hour for production. Below is the information they provided showing the hourly pay for all their laborers.

Laborer #	Per hour
1	$25.00
2	$26.00
3	$24.50
4	$15.00
5	$12.00
6	$12.00
7	$12.00
8	$12.00
9	$12.00
10	$22.50
11	$28.50
12	$27.25
13	$15.00
14	$24.60
15	$35.00

Questions:

A) Using the Basic Stats Tool, what minimum and maximum values, with 95% confidence, would you negotiate for laborer pay per hour?

B) What questions might you ask the company about the provided information? How could the answers to those questions change your negotiation values?

C) You are told that Laborers 5 thru 9 were summer hires and will not be working on the project. Does this change your negotiation values? If so, what new values will you use?

D) Can you think of information that may be better to use in order to estimate Labor costs on a contract?

Defense Acquisition University

CON 170

Block 2

Math Refresher

Attachment 2

Statistical Analysis Flowchart

Measures of Reliability

Since a sample contains only a portion of observation in the population, there is always a chance that our inference or generalization about the population will be inaccurate. Therefore, our inference should be accompanied by a measure of reliability

Measures of Central Tendency

Are values around which data values tend to cluster, they represent the central values of the distribution

Collect Data

Identifying Issues and Concerns

Questions to consider:

* Are the statistics representative of the current contract situation?
* Is your analysis, including any sample analysis, based on current, accurate and complete data?

Identifying Situations for Use

* Streamlining the evaluation of a large quantity of data without sacrificing quality. The underlying assumption that a sample is representative of the population from which it is drawn. Stratified sampling allows you to concentrate your effort

Mode

The value of the observation that occurs most often in the data set

Mean

$$\overline{X} = \frac{\Sigma X}{n}$$

Median

Middle value of a data set when the observation are arrayed from highest to the lowest

Developing Government Objectives
for contract Prices/Cost Based on Mean Values

Steps 1 and 2

Continued

Measures of Dispersion

* Though the mean of a data set is a value around which the other values tend to cluster, it conveys no indication of the closeness of this clustering (that is, the dispersion) indication of how closely these other values are clustered around the mean

Developing Negotiating Ranges around the Government Objective

Range

Highest - Lowest Observations

Variance

$$S^2 = \frac{\Sigma(X-\overline{X})^2}{n-1}$$

(See Notes 1&2)

Standard Deviation

$$S = \sqrt{S^2}$$

(See Notes 3& 4)

Measure of Relative Dispersion

* Comparison of the standard deviation of two or more sample distributions when the means are different (not equal) **Coefficient of Variation**

Standard error of the Mean

* Statistics, such as the mean ($\overline{X}$) or standard deviation (s), are computed on the basis of samples; hence, they are subject to sampling error. The distribution of all possible values for a statistic given a sample size n is called the sampling distribution of that statistic. The distribution of all possible values for the sample mean given a sample size n is called the sampling distribution of the mean. Given a normally distributed population: we use sample statistics to estimate these population parameters, That is

We use the sample mean and sample standard deviation to estimate the population mean and the population standard deviation

* **Critical Assumptions:** though the population mean and the population standard deviation are not normally known. **we do assume that cost or pricing data are normally distributed**

Standard Error of the Mean

$$S_{\overline{X}} = \frac{S}{\sqrt{n}}$$

Coefficient of Variation

$$CV = \frac{S}{\overline{X}}$$

Steps 3 and 4

t Distribution

1) Significance Level = 1- Confidence Level
2) **d.f. = n-1**
3) t Table

(See Note 5)

Confidence Level

Refers to the confidence that you have that a particular interval includes the population mean. A confidence interval for a population mean when the population standard deviation is known and n<30 can be constructed.

Identifying Situations for Use

* Developing an estimate of the range of risk surrounding the government objective for consideration in contract type selection

* Developing an estimate of the range of risk surrounding the government objective for consideration in profit/fee analysis

Confidence Level

$$\bar{X} \quad t\,S_{\bar{X}}$$

Identifying Issues and Concerns

* Have you considered the confidence interval in developing a range of reasonable costs?

* Is the confidence interval so large as to render the point estimate meaningless as a negotiation tool?

Steps 5 and 6

Notes:

Note 1:

Statistical adjustment for sample size: **n-1 example d.f. = degrees of freedom**. The use of **n-1** rather than n results in S^2 being a better estimator of the population variance. Samples are more alike than the population from which they are taken. Hence, the sample variance is likely to underestimate the true variation in the population. Division by **n-1** in a sense artificially inflates the sample variance but in so doing, it makes the sample variance a better estimator of the population variance.

Note 2:

Two shortcomings associated with the use of the **variance** as a measure of variation:

* Value is often large compared with the values of the data set
* Variance is in different denomination from the values in the data set (squared)

Note 3:

In the computation of the standard deviation (and variance) the differences between an observed value and the mean are squared. The technique in a sense imposes a higher penalty for those observations that are farther away from the mean.

Note 4:

Empirical Rule: Interval $\bar{X} \pm 1S$ includes approximately 68% of the total observations in the population

$\bar{X} \pm 2S$ includes approximately 95% of the total observations in the population

$\bar{X} \pm 3S$ includes approximately 99.7% of the total observations in the population

Note 5:

t-Distribution:

* Symmetrical but is a flatter distribution (higher in the tails) than the normal distribution
* Defined by d.f. (sample size less one, n-1)
* There is a different t distribution for each sample size
* As the sample size increases, the shape of the t-distribution approaches the shape of the normal distribution

Block 3

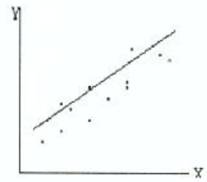
Introduction to Regression Analysis

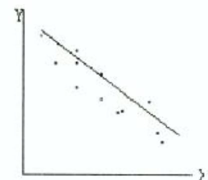
Overview

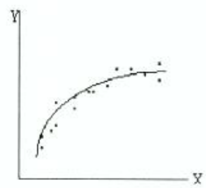
Introduction

In this block, we will discuss:

- The equation of a straight line.
- Linear Regression
- Variation in the Regression model
- Regression Analysis


Positive linear correlation


Negative linear correlation


Positive non-linear correlation


No correlation

Block objectives

At the conclusion of this block, you will be able to

- describe the parts of a straight line equation
- explain linear regression, and
- perform regression analysis.

In this unit

The following topics are located in this block:

Straight Lines

Straight lines and linear regression analysis

Everything you need to know about linear regression analysis you can learn from the equation of a straight line.

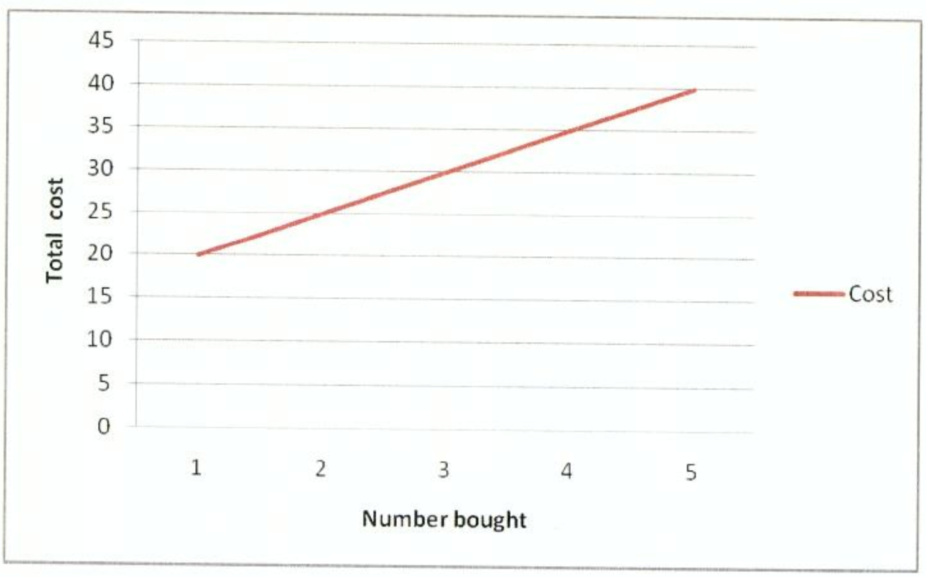
The straight line is

- one of the most commonly used and most valuable tools in both price and cost analysis, and
- useful for estimating relationships and projecting economic trends.

Equation of a straight line

The equation of a straight line is: $\mathbf{Y_c = A + BX}$

Y_c = Calculated Value of Dependant variable (Output)
A = Y intercept
B = Slope of the line
X = Independent variable (Input)



The graph above depicts part of the equation, Y = 15 + 5X. (From X = 1 to 5). The entire line continues on forever in both directions. Notice that you can predict what the Total Cost (Dependant Variable) will be for any X value (Independent Variable) you buy.

A) How much would you expect to pay if the Number bought is 4?

B) How much would you expect to pay if Number bought is 8?

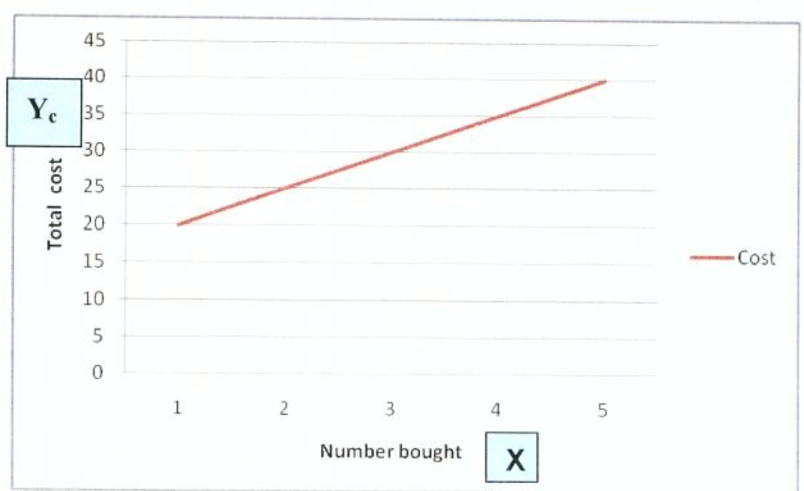
C) How much would you expect to pay if Number bought is 500? Would you trust this value to be accurate?

Straight Lines, Continued

Parts of the Straight Line Equation

The table below provides and explanation of the parts of a straight line equation.

Equation	Meaning of Equation
$\mathbf{Y_c}$	The 'Y_c' in the equation of a line is the 'answer' to the question "What will happen if the input X changes?" Thus the value of 'Y_c' is **dependent** on the value of 'X' In the example below, The calculated Total cost represents the 'Y_c' value and the Number bought represents the 'X' value.
X	The 'X' in the equation of a line is the value that is input to find 'Y_c' value. The 'X' is **independent** of the 'Y_c' value. In the example below any Number bought can be entered to find the corresponding Total cost. Of course not all 'X' values will make sense as you can not buy less than zero items. Also, 'X' values far outside of the given data will not produce accurate Y_c values. If the 'X' value input is invalid the output 'Y' will be invalid as well. It's garbage in, garbage out.
A	The 'A' in the equation of a line is where the line will cross the Y axis. This will only happen when the input value of 'X' is zero. Notice that in the equation Y = A + BX if the input 'X' is zero you are left with Y = A.
B	The 'B' in the equation of a line determines how steep the line is and if it moves in a positive or negative direction. The slope is calculated by dividing the rise of the line by the run (Rise over Run). In the example below when we 'run' along the x axis from 2 to 3 the line rises from 25 to 30. Thus the 'Y' value rises 5 for a run of 1 'X for a slope of 5/1 or 5.

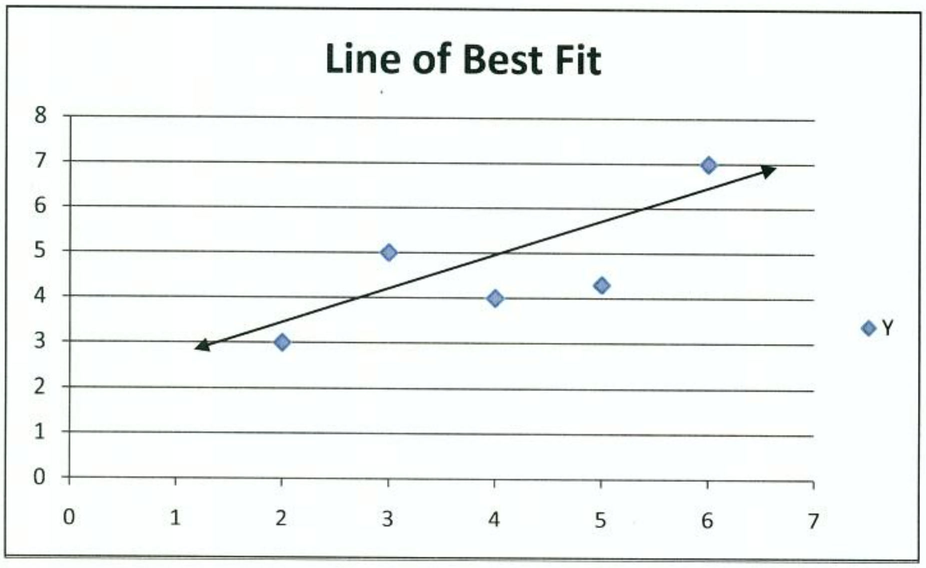


Note: In order to build the equation of a straight line only two points that lie on the line are needed.

Linear Regression

Linear Regression

Linear Regression is a method of constructing a line using a set of data points that do not all lay upon the same line. A line of best fit thru points that fall on the line can be done visually with a ruler and a graph but when the data points do not "line up" perfectly a mathematically precise method is used. This method is called Linear Regression or just Regression. The graph below depicts this. The purpose of regression analysis is to predict the value of a dependent variable given the value of an independent variable.



Use of Regression

Regression is used to:

- Develop and analyze price vs. cost estimates
- Analyze trends that appear related to time
- Forecasting wage rates, material costs, product prices

Independent and dependant variables

The formula for the line developed from linear regression will be the same as described above. The most important things to determine before implementing regression is which variable is independent and which is dependant. In this course it is usually safe to assume that cost or time is the dependant variable. For example, how much money it will cost to install a carpet **depends** upon how big the room is.

How good (strong) is the relationship?

Linear regression can be used to make an estimating relationship between any two variables. For example, you can make cost estimations between the color of a computer monitor (independent variable) and its price (dependant variable) but it would not be a strong estimate as the relationship between a monitor's color and it's cost is weak or non existent. A better estimate could be made by comparing monitor size with cost.

Variation in the Regression Model

Variation in the Regression Model

In order to determine how good the estimating equation is we must determine how much variation exists in the model and partition it into explained variation and unexplained variation. (See handout "Trend/Regression Analysis Flowchart" for more details.)

SSR

The sum of squares regression (SSR) is a measure of variation of Y_c that is explained by the regression equation. SSR is the sum of the squared differences between the calculated value of Y (Y_c) and the mean of Y.

SSE

The sum of squares error (SSE) is a measure of variation of Y_c that is not explained by the regression equation. SSE is the sum of the squared differences between the observed values of Y (Y_i) and the calculated value of Y (Y_c).

SST

The sum of squares total (SST) is a measure of the total variation of Y. The SST is the sum of SSR and SSE. Thus SST = SSR + SSE.

Analysis of the Variation

Once the amount of variation in the regression model is known we can analyze it to determine the variance of the model. Variance is equal to the variation divided by the degrees of freedom (df). In regression analysis, df is a statistical concept that is used to adjust for bias in the sample in estimating the population mean.

MSR

The Mean Square Regression (MSR) will equal the sum of squares regression (SSR) divided by the df. For simple linear regression the df for MSR will always be 1. This means the MSR will equal the SSR.

MSE

The Mean Square Error (MSE) will equal the sum of squares error (SSE) divided by df. In simple linear regression the df for MSE will always be n-2 where n = the number of samples.

SEE

The Standard Error of the Estimate (SEE) is a measure of the accuracy of the regression equation. The SEE is equal to the square root of the MSE. The calculated Y (Y_c) +/- 3* SEE will contain approximately 99% of the total observations.

Analyzing the Regression Model

Strength of relationship

In regression analysis the strength of the relationship is determined by the **coefficient of determination**, which is called r^2. The formula for r^2 is SSR/SST.

The coefficient of determination is always a number between 0 and 1.

The closer r^2 is to 1, the stronger the relationship. If r^2 is near 0 it indicates little or no relationship.

Analyzing r^2

The coefficient of determination is the ratio of explained variation to the total variation. It measures how much variation is due to the independent variable vs. random error.

For example, an r^2 of .90 indicates 90% of the variation in Y is due to its relationship with X. This means 90% of the variation is explained by the regression line.

By convention, an $r^2 \geq .8$ is considered a good relationship.

Statistical significance

Once r^2 shows us we have a good relationship between the dependant and independent variables we need to determine if the relationship is statistically significant.

To do this we must calculate a 'T' value and compare it to a 't' value from a "t" distribution table. This will be done for us using an Excel spreadsheet.

If the equation is . . .	**The relationship is. . .**
$T > t$	statistically significant
$T = t$	indifferent
$T < t$	not statistically significant

Note: > = "greater than" sign   < = "less than" sign

If the relationship is not statistically significant the regression formula should not be used to make predictions.

When to Use Regression Analysis

When to use Regression Analysis

Regression analysis is used to

- Establish cost estimating relationships between independent variables and cost/price.
- Quantify the relationship between the indirect cost rate base and pool over time. If one can quantify the relationship, one can use it to develop and analyze indirect cost rate estimates.
- Analyze price trends that appear to be related to time if other economic factors that drive price cannot be used. If used to predict trends over time short term analysis is more accurate than long term. (It is easier to predict your work schedule for next week than for two months from now.)

Steps for Trend/Regression Analysis

Regression analysis is done by completing these 11 'simple' steps.

Step	Action
1	Collect data.
2	Graph data. Does there appear to be a linear relationship?
3	Calculate the sum values of: X, Y, XY, X^2, and Y^2
4	Compute the Mean of X and the Mean of Y.
5	Compute the slope B and the Y intercept A.
6	Develop the estimating equation Y = A + BX
7	Use Analysis of Variation (ANOVA) table. SST = SSR + SSE
8	Determine the strength of the relationship (r^2)
9	Use the T-test to determine if the relationship between X and Y is significant.
10	Compute the Standard Error of the Estimate
11	Compute the Prediction Interval (Y +/-)

Good news

In class we will use an Excel spreadsheet (file name: **Regression-Tool.xlsx)** to perform the linear regression.

We will analyze the output to make predictions and estimates.

Using Excel to Perform Regression Analysis

Using the Linear Regression Tool

We will now use the Regression Tool to estimate the cost of installing carpet in a 300 square foot area. Open the Regression Tool spreadsheet **Regression-Tool.xls.**

Notice the blue and yellow cells. These are the cells that data can be entered in. There is a column for Dependent (Y) values and Independent (X) values.

1. Place your mouse pointer over the cells to see comments explaining what goes in each cell.
2. Input the following historical values in the Dependent (Y) and Independent (X) columns.

Cost	Sq. Footage
1000	250
989	225
1850	400
1500	320

Notice that as you add data points they are plotted on the graph to the right and a best fit line is drawn.

Does it seem as if the data entered has a linear relationship? If it does, then linear regression may be used to estimate the cost of installing carpet for Sq. Footage's not given in the table.

Exercise

Follow the steps below to complete the Regression Tool exercise.

Step	Action
1	Scroll down on the sheet and enter "300" in the yellow square for the Sq. Footage for which you desire an estimate. What is your estimated cost for 300 Sq. Feet?
2	Look at the Estimating Formula. Note: The Estimating Formula the equation of the line that is being used in this problem. What is the Y intercept of the equation? What is the slope of the line?
3	Input a Confidence Level of 90% in the space provided. Put your mouse over the comments cell to see what this is. With a Confidence Level of 90% what is your low to high estimate for the cost of 300 Sq. Feet?

Using Excel to Perform Regression Analysis, Continued

Exercise, continued

Follow the steps below to complete the Regression Tool exercise.

<table>
<tr><th>Step</th><th>Action</th></tr>
<tr><td>4</td><td>Look at the Goodness-of-Fit Statistics.

Put your mouse over each comment to see what each value represents.

The most important value to make note of is the R-Squared value.

<table>
<tr><th>If R-Squared is. . .</th><th>Then. . .</th></tr>
<tr><td>over 80%</td><td>the strength of association between the Dependent and Independent values is considered strong.</td></tr>
<tr><td>below 80%</td><td>they are not well associated and other data should be used.</td></tr>
</table>
</td></tr>
<tr><td>5</td><td>Look at the Analysis of Variance (ANOVA) Table.

Put your mouse over each comment to read what each value represents.

The most important value to make note of is the T-Statistic for Sq. Footage.
In this case it is 9.63.

Compare this number to the t table value with Level of confidence of 90%.
In this case it is 2.92.

<table>
<tr><th>If. . .</th><th>Then. . .</th></tr>
<tr><td>T > t</td><td>the relationship between X and Y is statistically significant and the equation can be used.</td></tr>
<tr><td>T < t</td><td>there is no significant relationship and the equation should not be used.
In this case T > t so the equation can be used</td></tr>
</table>
</td></tr>
</table>

Using Excel to Perform Regression Analysis, Continued

Scenario 1: Cost for thrust

You believe there is a Cost Estimating Relationship (CER) between the thrust of a jet engine and its cost. You have gathered data to test your hypothesis. Use the regression analysis tool to estimate the cost of an engine that produces 12,000 pounds of thrust. The **Independent** (X) value is the Thrust, the **Dependent** (Y) value is the Unit Cost.

Model	Unit cost ($1,000's)	Thrust (lbs, 1,000's)
A	50	5.2
B	70	6.3
C	160	9.1
D	140	7.5
E	225	13

A) Does there appear to be a linear relationship between Cost and Thrust?

B) What equation for a line was generated? (Estimating Formula on sheet)

C) What is the 'Y' intercept of the line?

What is the Slope of the line?

D) Input the value 12,000 in the estimate box.

What is the estimated unit cost for an engine with 12,000 pounds of thrust?

E) With a Confidence Level of 90% what is the prediction interval?
(Low to High, (+/-) divided by Range.)

F) Does the data fit the line well?
(Goodness-of-Fit above 80%? This is the R-Squared value on the sheet.)

G) Is the relationship statistically significant?
(Compare T-Statistic in box with the Table "t" statistic under 90%. If T > t the relationship is statistically significant.)

Using Excel to Perform Regression Analysis, Continued

Scenario 2: Motorcycle Top Speed vs. Cost

You believe there is a Cost Estimating Relationship (CER) between the top speed of a motorcycle and its cost. You have gathered data to test your hypothesis. Use the regression analysis tool to estimate the cost of a motorcycle with a top speed of 120 mph.

Motorcycle	Top Speed (MPH)	Cost
A	95	$20,000.00
B	110	$16,000.00
C	165	$14,000.00
D	100	$7,200.00
E	125	$9,000.00
F	200	$24,000.00
G	150	$8,500.00

A) Does there appear to be a linear relationship between Cost and Top Speed?

B) What equation for a line was generated?

C) What is the 'Y' intercept of the line?

What is the Slope of the line?

D) Input the value 120 in the estimate box.
What is the estimated unit cost for bike with a top speed of 120 MPH?

E) With a Confidence Level of 90% what is the prediction interval?

Does this make sense?

F) Does the data fit the line well?
(R-Squared above 80%)

G) Is the relationship statistically significant?
(T > t)

H) Do you think using a motorcycle's top speed is a good estimate of its cost?

Using Excel to Perform Regression Analysis, Continued

Scenario 3: Engineering Labor Cost Estimate

You have asked the company Radar-Love to send you historical data on their mean hourly engineering labor rates from the previous five years. They have sent you the following data. Use regression analysis with a 90% confidence level to complete the table.

	2005	2006	2007	2008	2009	Est. 2010	Est. 2011	r^2	T > t?
Design	$52.00	$54.25	$54.30	$55.50	$56.20				
Testing	$48.50	$49.25	$50.00	$51.25	$52.00				
Proto-typing	$28.00	$28.00	$28.50	$28.75	$29.10				

A) Does there appear to be a linear relationship between engineering labor rates and time?

B) Does the data from each disciple fit the line well?
(R-Squared above 80%)

C) Are the relationships statistically significant?
(T > t)

D) With a confidence level of 90% what would you expect to pay the company for their total engineering efforts in 2011?

Using Excel to Perform Regression Analysis, Continued

Scenario 4: Laborer Cost Estimate

You have asked the company Radar-Love to send you historical data on their mean hourly laborer pay rates from the previous five years. They have sent you the following data. Use regression analysis with a 90% confidence level to complete the table.

	2005	2006	2007	2008	2009	Est. 2010	Est. 2011	r^2	T > t?
Manufacture	$22.50	$23.10	$24.25	$25.00	$25.65				
Assemble	$10.00	$10.10	$10.25	$10.50	$10.75				
Quality control	$17.80	$17.50	$17.35	$16.90	$16.25				

A) Does there appear to be a linear relationship between laborer pay rates and time?

B) What is 'different' about the Quality control regression line?

What is its slope?

C) Does the data from each disciple fit the line well?
(R-Squared above 80%)

D) Are the relationships statistically significant?
(T > t)

E) With a confidence level of 90% what would you expect to pay the company for their total laborer efforts in 2011?

Using Excel to Perform Regression Analysis, Continued

Scenario 5: Indirect cost rates

The formula to determine indirect cost rates is:

$$Indirect\ Cost\ Rate = \frac{\text{Indirect Cost Pool}}{\text{Indirect Cost Allocation Base}}$$

In the formula the Pool is the dependent Variable and the Base is the independent variable. Use the regression tool to complete the following tables. Make sure the data meets both the goodness-of-fit and statistical significance tests.

Account	2008		2009		2010		2011	
	Pool $	Base $	Pool $	Base $	Pool $	Base $	Pool $	Base $
Material	290.5	7500	312	8000	270	7000		8250
Engineering	3082	6650	2750	6050	2900	6250		6582
G&A	5615	46000	6589	55000	6000	50500		65000

Overhead Rates	2008	2009	2010	2011
Material	3.87%	3.9%	3.86%	
Engineering	46.35%	45.45%	46.4%	
G&A	12.21%	11.98%	11.88%	

A) Does the data meet both the goodness-of-fit and statistical significance tests?

B) What happens to the overhead rate when the size of the base is reduced?

C) What happens to the overhead rate when the size of the base is increased?

Defense Acquisition University

CON 170

Block 3
Math Refresher
Attachment

Regression Flowchart Steps

Trend/Regression Analysis

Step 1: Collect Data = n

Step 2: Graph Data, Is there a linear relationship?

Step 3: Calculate the sum values of:

$$X, Y, XY, X^2, Y^2$$

Step 4: Compute the mean of $\overline{X}$ and the mean of $\overline{Y}$

Step 5: Compute B the slope

Step 6: Compute A the Y intercept

Step 7: Develop the estimating equation $Y_c = A + BX$

Step 8: Analysis of Variation ANOVA table

$$SST = SSR + SSE$$

Step 9: Determine the strength of the relationship (r^2) between X and Y

Step 10: Is the relationship between X and Y significant? The T-test:

$T > t$ Yes

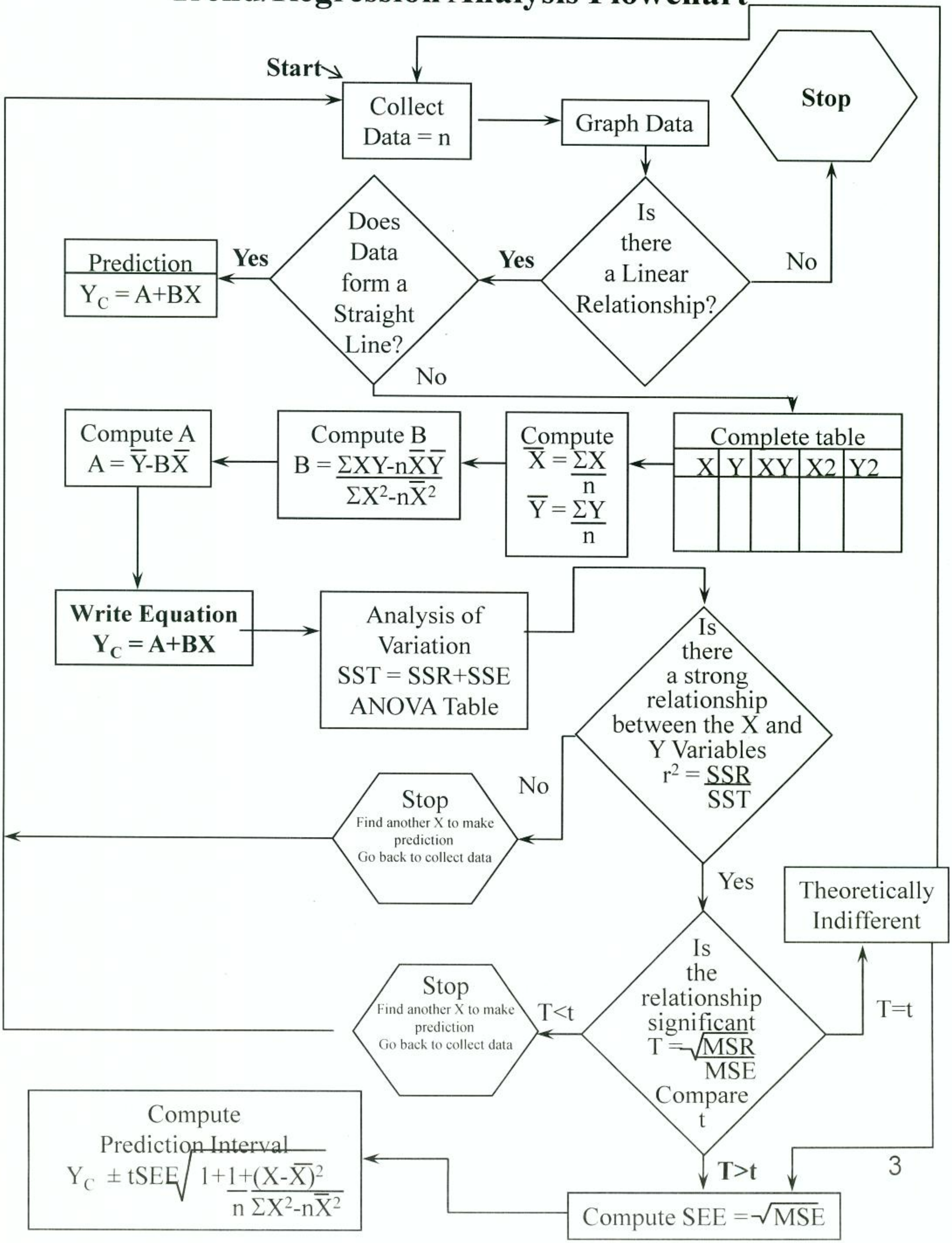
$T < t$ No

$T = t$ Indifferent

Step 11: Compute the Standard Error of the Estimate (SEE)

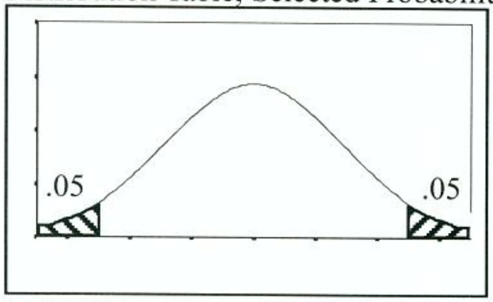
Step 12: Compute Prediction Interval $Y_c \pm$

Trend/Regression Analysis Flowchart



Appendix B

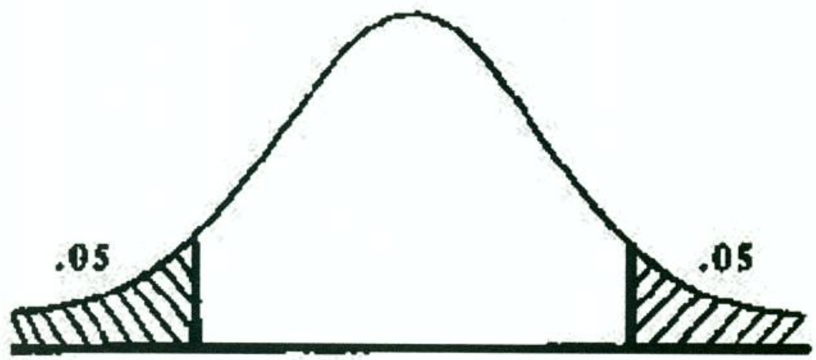
t Distribution Table, Selected Probabilities



	Significance Level (both tails combined)			
d.f.	.10	.05	.02	.01
1	6.314	12.706	31.821	63.657
2	2.920	4.303	6.965	9.925
3	2.353	3.182	4.541	5.841
4	2.132	2.776	3.747	4.604
5	2.015	2.251	3.365	4.032
6	1.943	2.447	3.143	3.707
7	1.895	2.365	2.998	3.499
8	1.860	2.306	2.896	3.355
9	1.833	2.262	2.821	3.250
10	1.812	2.228	2.764	3.169
11	1.796	2.201	2.718	3.106
12	1.782	2.179	2.681	3.055
13	1.771	2.160	2.650	3.012
14	1.761	2.145	2.624	2.977
15	1.753	2.131	2.602	2.947
16	1.746	2.120	2.583	2.921
17	1.740	2.110	2.567	2.898
18	1.734	2.101	2.552	2.878
19	1.729	2.093	2.539	2.861
20	1.725	2.086	2.528	2.845
21	1.721	2.080	2.518	2.831
22	1.717	2.074	2.508	2.819
23	1.714	2.069	2.500	2.807
24	1.711	2.064	2.492	2.797
25	1.708	2.060	2.485	2.787
26	1.706	2.056	2.479	2.779
27	1.703	2.052	2.473	2.771
28	1.701	2.048	2.467	2.763
29	1.699	2.045	2.462	2.756
30	1.697	2.042	2.457	2.750
40	1.684	2.021	2.423	2.704
60	1.671	2.000	2.390	2.660
120	1.658	1.980	2.358	2.617

CON 170 Math Refresher
Block 3
Attachment

DISTRIBUTION TABLE, SELECTED PROBABILITIES



Example: degrees of freedom = 14; significance level = .10

Significance Level (both tails combined) DISTRIBUTION TABLE, SELECTED PROBABILITIES

d.f.	.10	.05	.02	.01
1	6.314	12.706	31.821	63.657
2	2.920	4.303	6.965	9.925
3	2.353	3.182	4.541	5.841
4	2.132	2.776	3.747	4.604
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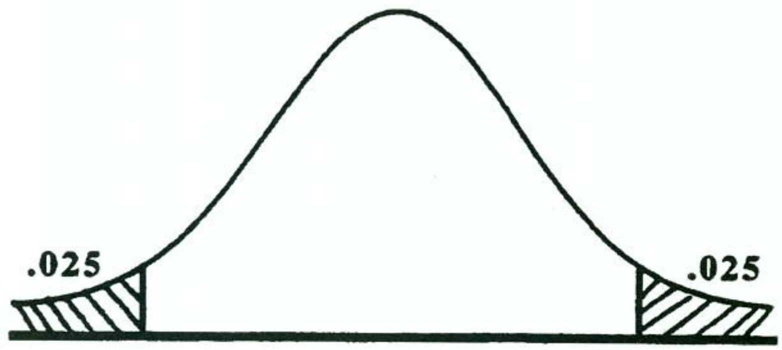
17	1.740	2.110	2.567	2.898
18	1.734	2.101	2.552	2.878
19	1.729	2.093	2.539	2.861
20	1.725	2.086	2.528	2.845



Example: degrees of freedom = 24; significance level = .05

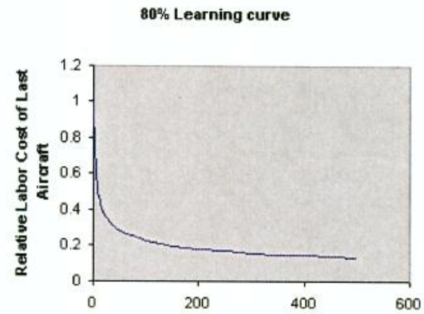
Significance Level (both tails combined)

d.f.	.10	.05	.02	.01
21	1.721	2.080	2.518	2.831
22	1.717	2.074	2.508	2.819
23	1.714	2.069	2.500	2.807
24	1.711	2.064	2.492	2.797
25	1.708	2.060	2.485	2.787
26	1.706	2.056	2.479	2.779
27	1.703	2.052	2.473	2.771
28	1.701	2.048	2.467	2.763
29	1.699	2.045	2.462	2.756
30	1.697	2.042	2.457	2.750
40	1.684	2.021	2.423	2.704
60	1.671	2.000	2.390	2.660
120	1.658	1.980	2.358	2.617

Block 4

Improvement Curve

Overview



Introduction

In this block, we will discuss:

- Features of an improvement curve
- Improvement curve terms and definitions
- Improvement curve software

Block objective

At the conclusion of this block, you will be able to determine when improvement curves should be used, analyze their results and determine if those results are valid.

In this block

The following topics are located in this block:

Improvement Curves

Improvement Curve

The improvement curve (also known as the Crawford curve and Boeing curve) is based on the concept that, as a task is performed repetitively, the time required to perform the task will decrease. You may have experienced this effect yourself.

Use of the improvement curve in Government and industry

Since World War II, the improvement curve concept has been used by Government and industry to aid in estimating labor costs when preparing contracts. Over the years, the improvement curve has been used as a contract estimating and analysis tool in a variety of industries including:

- airframes
- electronics systems
- machine tools
- shipbuilding
- missile systems, and
- depot level maintenance of equipment.

Rate of Improvement (ROI)

The improvement curve theory states that as the total volume of units produced doubles, the cost in time spent per unit decreases by some constant percentage. The constant percentage by which the costs of doubled quantities decrease can be calculated and is called the Rate of Improvement (ROI). As a result, improvement curves can be used to estimate contract price, direct labor-hours, direct material cost, or many other recurring contract costs.

Calculating Rate of Improvement (ROI)

One way to calculate the ROI is to determine the percent change in time to make lots. This method was demonstrated in unit one and is shown in Unit 1 on page 24.

The formula to calculate the ROI is:

$$ROI = \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} * 100$$

The ROI will be a negative number. This is because the slope of the improvement curve is negative.

Slope

The term "slope" in the improvement curve analysis is the difference between 100 % and the **absolute value** of the rate of improvement. If the rate of improvement is 20%, the improvement curve slope is 80% (100% – 20%). This means that the larger the ROI the smaller the slope and vice versa.

Given the slope, the ROI can be calculated as 100% minus the slope.

Improvement Curves, Continued

Unit improvement curve

The unit improvement curve theory was a model validated by a Stanford Research Institute study. This theory can be stated as:

As the total volume of units produced doubles the cost per unit decreases by some constant percentage.

The equation for the unit improvement curve is expressed as:

$$Y = AX^B$$

Where:

Y = the unit cost of the X^{th} unit.
A = the theoretical cost of the first unit. (Sometimes called T_1)
X = the unit number

$$B = \frac{\text{Logarithm of the Slope}}{\text{Logarithm of 2}}$$

Cumulative average improvement curve

The cumulative average improvement curve was introduced in 1936. This theory can be stated as:

As the total volume of units produced doubles the ***average*** *cost per unit decreases by some constant percentage.*

The equation for the cumulative average improvement curve is the same as the unit improvement curve shown above with one difference which is:

Y = Cumulative average unit cost through the X^{th} unit.

The difference between the curves

The biggest difference between the unit improvement curve and the cumulative average improvement curve is found in the first few units of production. Over the first few units of production an operation following the cumulative average curve will experience a much greater reduction in cost than an operation following the unit improvement curve with the same slope.

Because of this, it is generally considered appropriate to use the unit improvement curve when a company is fully prepared for production and the cumulative average curve when a company is not completely ready for production. This is because it is expected that an unready company will improve much faster as they are expected to be somewhat inefficient to begin with. In later production, the reduction in cost for operations using either curve will be about the same.

In practice, companies typically use one type of curve regardless of their production situation. Most firms in the airframe industry use the cumulative average curve while most firms in other industries use the unit curve. In this course we will use the unit improvement curve.

Improvement Curves, Continued

When to use the improvement curve

The improvement curve cannot be used as an estimating tool in every situation. Use of the curve should be considered in situations were there is:

- A high proportion of manual labor
- Uninterrupted production
- Production of complex items
- No major technological changes
- Continuous pressure to improve

Analyzing improvement using the unit improvement curve

To illustrate the effect of the unit improvement curve, let us assume that the first unit of some complex item required 100,000 labor-hours to produce.

If the slope of the improvement curve is 80%, the following table demonstrates the labor-hours required to produce units at successively doubled quantities.

Units Produced	Labor hours per unit	Difference in labor hours per unit (Time saved)	Rate of improvement	Slope of curve
1	100,000		20%	80%
2	80,000	20,000	20%	80%
4	64,000	16,000	20%	80%
8	51,200	12,800	20%	80%
16	40,960	10,240	20%	80%
32	32,768	8,192	20%	80%

<u>Note</u>: Can you build this table in Excel such that only the initial labor hours per unit and the initial rate of improvement are entered and the rest of the data self populates?

You can see that the:

- **Amount of labor-hour reduction between doubled quantities is not constant**
  The number of hours of labor being 'saved' is constantly declining however, the rate of change remains constant at 20%.

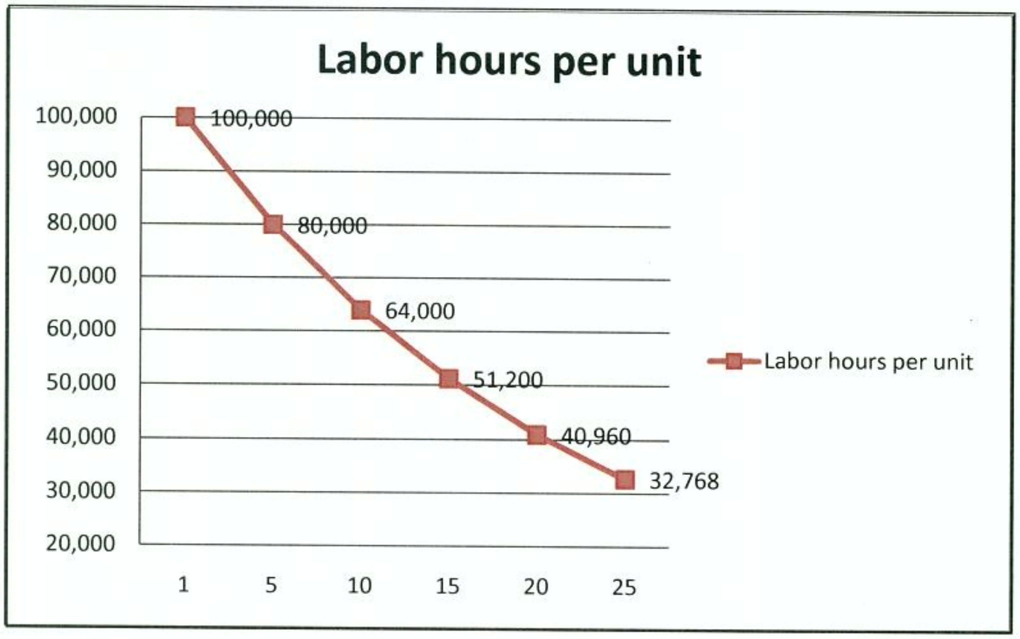
- **Number of units required to double the quantity produced is constantly increasing**
  It takes only one unit to double the quantity produced from unit 1 to unit 2 with a 'savings' of 20,000 hours.

  But it takes 16 units between unit 16 and unit 32 with a savings of only 8,192.

Improvement Curves, Continued

The improvement curve graph

The Labor-Hours graph of this data drawn on ordinary graph paper becomes a curve as shown on the graph below.



The graph curves because the number of hours of reduction between doubled quantities is constantly declining and an increasing number of units are required to double the quantity produced.

Note that most of the improvement takes place early in production. The curve will become almost flat as production continues and will reach a point where no further improvement occurs. At this point the standard time of production has been reached and the improvement curve can no longer be used to estimate future lot costs.

Although this graph is accurate it is difficult to make a good prediction of future costs from the curve. To solve this problem, such data is usually plotted on full-logarithmic graph paper. On such paper the data will fall in nearly a straight line and linear regression can be used to predict the labor hours required for the next data point. We do not use full-logarithmic graph paper in this class but will use Excel software to make the predictions.

The calculations for the improvement curve

When we discuss improvement curves we discuss them in terms of the theoretical value for unit #1 and the slope of the curve. With these two values we can graph the curve on paper or with computer programs to estimate the cost of future units.

The value of unit #1 is referred to as a theoretical value (T_1) because in most cases the actual cost of unit #1 is unknown. The slope of the curve equals 100% minus the Rate of Improvement (ROI). In this course historical data and a spread sheet program will be used to calculate the cost of t_1 future units.

Improvement Curve Terminology

The coefficient of determination, r^2

As you recall from unit 3 of this Participant Guide, in regression analysis the strength of the relationship is determined by the coefficient of determination, r^2. The coefficient of determination is always a number between 0 and 1. The closer r^2 is to 1, the stronger the relationship. If r^2 is near 0 it indicates little or no relationship. This same rule applies to the improvement curve values. By convention if $r^2 \geq .80\%$ it is considered a good relationship.

The Coefficient of Variation (CV)

As you recall from unit 2 of this Participant Guide, the CV is the measure of relative dispersion of a data set when the means of the data sets are not the same. CV is obtained by dividing the standard deviation by the mean. The CV allows the comparison of apples to oranges. The smaller the CV the less variation is in the data. While using improvement curves the lower the CV the better.

The Average Unit Cost (AUC)

The AUC is calculated by dividing the total amount of time it took to produce a particular lot by the size of the lot.

For example, if a lot of 155 units is produced in 1,700 hours the AUC is calculated as 1,700/155 = 10.96 or about 11 hours per unit.

The Algebraic Lot Midpoint (LMP)

The LMP is the unit in the lot whose actual cost/hours is equal to the average cost/hours of all the units in the lot. The LMP satisfies the equation:

Unit Cost/Hours * Lot Size = Lot Cost/Lot Hours.

The LMP can be approximated by using the following equations:

Lot 1 size < 10: LMP = Lot Size/2
Lot 1 size > 10: LMP = Lot Size/3

All other lots: $LMP = \frac{(F+L)+2\sqrt{F \cdot L}}{4}$

Where:
F = First unit number of the lot
L = Last unit number of the lot

In this course the LMP will be calculated using software.

Lot size

To calculate the size of a given lot subtract the first unit number from the last unit number and add 1. For example, consider a lot containing units 49 thru 203. Since 203 – 49 + 1 = 155 there are 155 units in the lot.

Improvement Curve Analysis Software

Using the Improvement Curve software tool

Now that you have become familiar with the concepts and terms of the improvement curve we are going to open and use a pre-made Excel spreadsheet to analyze improvement curves. You should have available to you a spreadsheet with the file name: Con 217 Improvement Curve Tool.xls. Find this file and double click it to open it. We will discuss how to enter data and how to interpret the results and then work on the following scenarios.

Scenario 1: Cost for aircraft parts

The Navy has ordered 50 aircraft parts and you need to estimate the time it will take the manufacturer to produce them. Historical data is shown below. Calculate the number for the first and last unit for each lot and use the Improvement Curve Tool to answer the questions below.

Lot number	Number of units in the Lot	Labor hours to produce the lot
1	46	2400
2	51	2000
3	100	3200
4	52	1300
5	50	?

A) How much time do you estimate it will take to produce lot #5?

B) How much time did the First Unit (T_1) take to make?

C) What is the Slope of the improvement curve?

D) What is the Rate of Improvement (ROI)?

E) What is the coefficient of determination (r^2)?

Is the prediction acceptable?

F) What is the Average Unit Cost (AUC) for the 46 units in Lot #1?

G) What is the Average Unit Cost (AUC) for lot #5?

H) What percentage change occurred between the AUC for Lot #1 and Lot #5?

I) If a 6^{th} lot of 45 units is ordered how many labor hours will be needed to produce it?

Improvement Curve Analysis Software, Continued

Scenario 2: Balancing act

Mass Balance Inc. is bidding on a solicitation to supply a new type of large digital scale for use at truck weighing stations in Afghanistan. DLA has an order is for 40 units. The first scale is predicted to take 60 hours of direct labor to produce and the second scale will take 48 hours to produce. Answer the following questions: (Use the 'Quick Calculator" portion of the Improvement Curve Tool.)

A) What is the Rate of Improvement?
(Second scale time minus First scale time)/First scale time * 100

B) What is the Slope of the curve?

C) What is the estimated time needed to produce the 40th unit?

D) What is the estimated time for to produce all 40 units?

E) What is the average time per unit for producing the last 10 units?

F) Mass Balance Inc.'s estimated a direct labor cost of 1,000 hours to produce all 40 units.

Is this a feasible estimate?

Improvement Curve Analysis Software, Continued

Scenario 3: Green Freeze

A large refrigerant company is bidding on a solicitation to convert DLA refrigerator trucks to use CFC-free refrigerant. The company has developed the following delivery schedule.

Week	Units
1	20
2	65
3	100
4	140
5	120

Historically, the learning rate for such projects has been 90%. The price the bidder anticipates for the contract includes the cost of 40 direct labor workers working 40 hours per week to meet this schedule. The first unit is predicted to take 30 hours to convert. Answer the following questions:

A) How many direct labor hours are available each week?

B) How many direct labor hours are available for the entire 5 weeks?

C) What is the total lot cost for the conversion of all the trucks?

D) What is the lot cost of lot #1?

What is the Average Unit Cost (AUC) of lot #1?

E) What is the lot cost of lot #5?

What is the Average Unit Cost (AUC) of lot #5?

F) What is the Mean AUC cost for the entire contract?

G) Is the use of this improvement curve valid?

H) Does the contractor's anticipated labor cost seem reasonable?

I) If another 100 trucks are to be converted in week 6 what would its lot cost AUC be?

Improvement Curve Analysis Software, Continued

Scenario 4: Some Assembly Required

A large assembly shop has bid on a solicitation to provide 500 units of a complex new part. The bid price is based on an average (Mean) of 20 hours of direct labor per unit. When the shop built the two required First Article Test (FAT) units the first unit took 50 hours to complete and the second took 45 hours to compete. Assuming the shop will build the 500 units in lots of 100 each answer the following questions:

A) What is the Rate of Improvement (ROI) for this process?

B) What is the slope of the improvement curve?

C) How many hours do you predict it will it take to make the third unit?

D) Using your answer for question C as T_1 how much time do you predict it will take to produce the first 100 units?
(The FAT units do not count as production units.)

E) How many hours do you predict it will take for the company to produce all 500 units?

F) What is the Lot Mid-Point (LMP) and the AUC for the third production lot?

G) Does the average of 20 hours of direct labor per unit included in the bid price seem reasonable?

H) If a 6^{th} lot 100 units is made how many direct labor hours will be required?

I) The solicitation is modified reducing the request to 250 units delivered in three lots with the final lot having 50 units. Will the average of 20 hours of direct labor per unit in the bid price be reasonable?

Improvement Curve Analysis Software, Continued

Scenario 5: Have a Seat

SDASU Inc. is bidding on a solicitation to fabricate and assemble 200 MRAP seats in two lots of 100 each. The seats are complex and considered critical safety items so higher level quality requirements are required. SDASU's bid proposed fabrication will require 9 hours per unit, assembly will require 2.5 hours per unit, and Quality Control will require 0.7 hours per unit. The company has produced the seats in the past and has sent the following historical data to substantiate their direct labor costs. Use the Improvement Curve Tool to analyze the data and answer the questions below.

Lot #	Quantity	Fabrication hours	Assembly hours	Quality hours
1	25	350	105	24
2	25	300	80	23
3	50	450	120	35
4	100	700	225	58
total	200	1800	530	140
Average		**9**	**2.5**	**0.7**

A) How did SDASU calculate their average direct labor costs?

Do you think this a valid method?

B) What is the slope of the fabrication improvement curve?

Is the use of the curve valid?

C) What do you estimate the AUC will be for the fabrication of lots #5 and #6?

D) What is the slope of the assembly improvement curve?

Is the use of the curve valid?

E) What do you estimate the AUC will be for the assembly of lots #5 and #6?

F) What is the slope of the quality improvement curve?

Is the use of the curve valid?

G) What do you estimate the AUC will be for the quality inspections of lots #5 and #6?